Bonus Confidence Intervals

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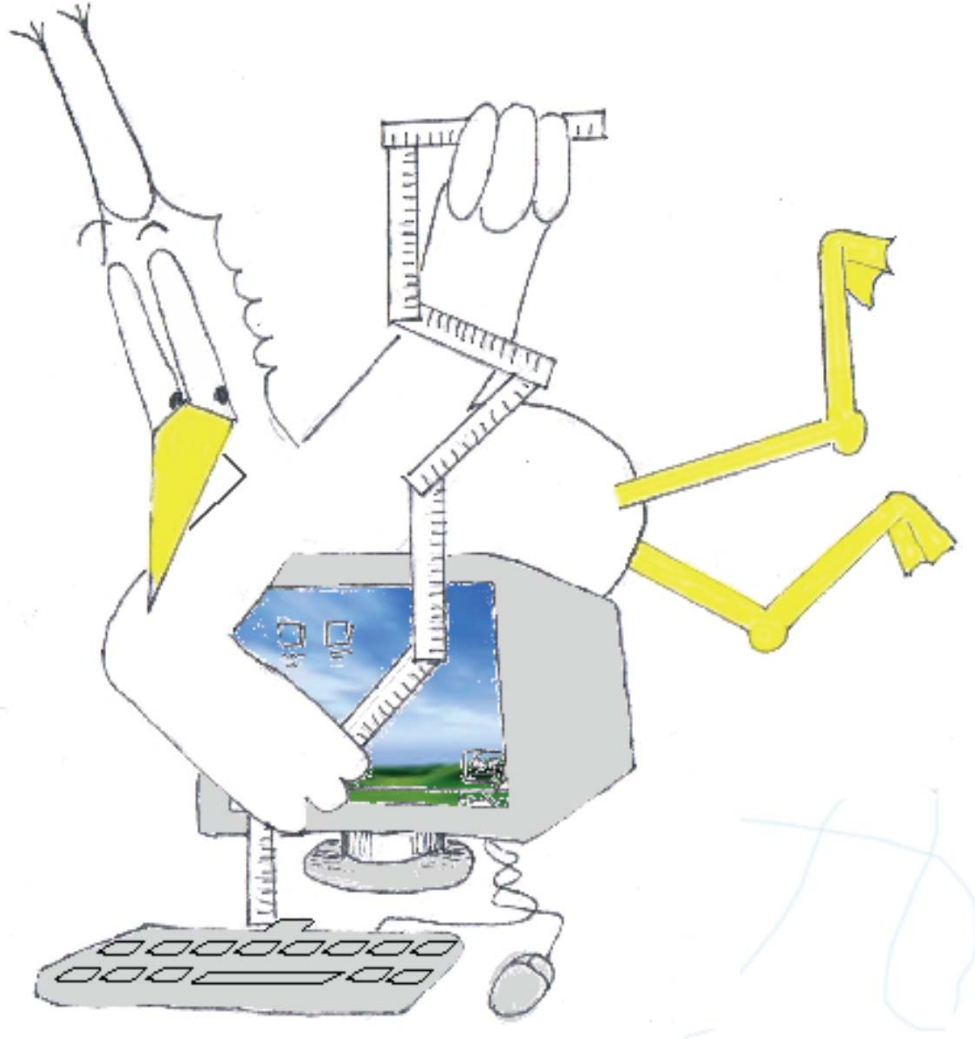


Here are the results of 10 independent simulation runs (throughput in Mb/s).

13.3

12.9

13.1

13.5

12.8

12.9

13.2

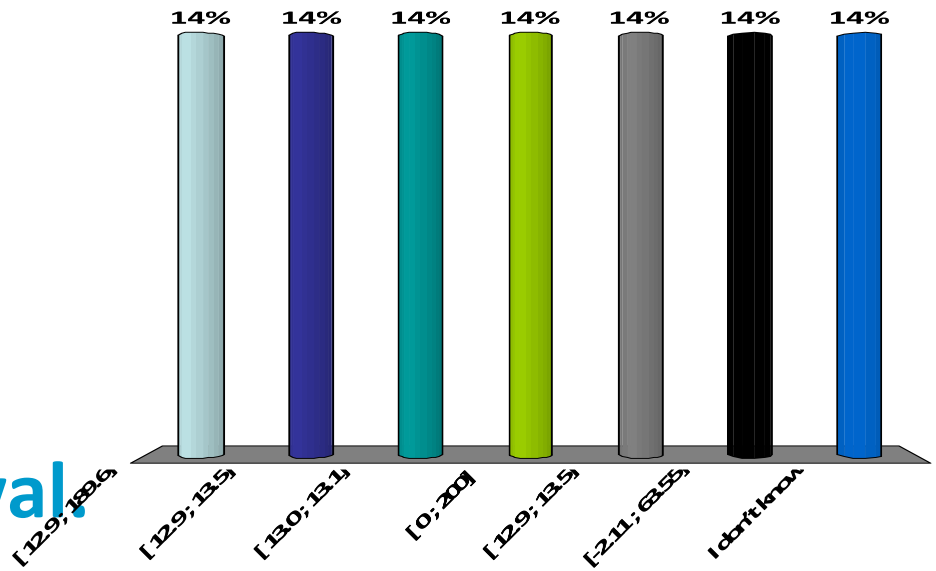
189.6

12.9

13.0

Give a 95% confidence interval.

A. [12.9 ; 189.6]
B. [12.9 ; 13.5]
C. [13.0 ; 13.1]
D. [0 ; 200]
E. [12.9 ; 63.55]
F. [-2.11 ; 63.55]
G. I don’t know



Solution

Method 1: CI for median

$$\left[x_{(2)}\,;x_{(9)}\right] = [12.9\,;13.5]$$

Method 2: CI for mean $m \pm 1.96\,\frac{s}{\sqrt{n}}$

mean $m = 30.72$

Standard deviation $s = 52.96$ with $\frac{1}{n}$ formula

Standard deviation $s = 55.83$ with $\frac{1}{n-1}$ formula

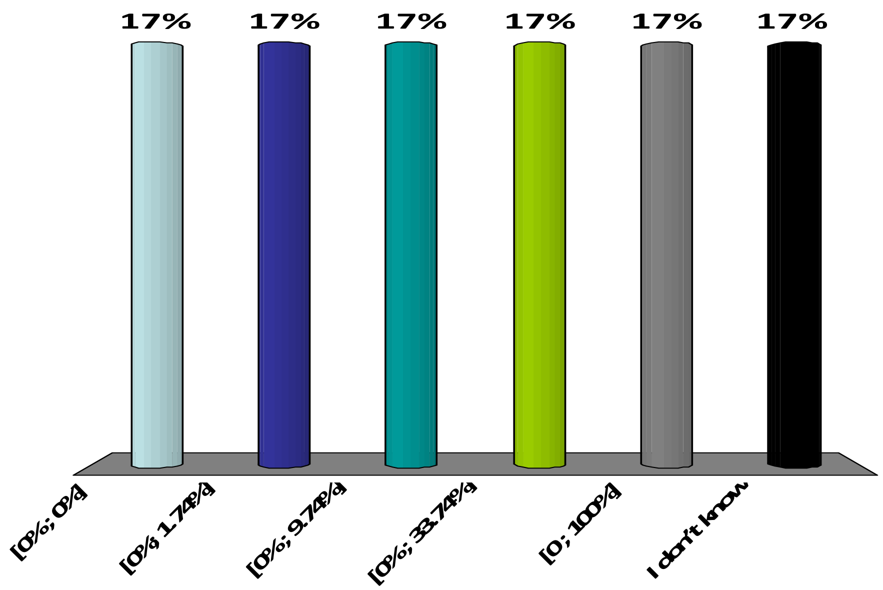
CI $= [-2.11\,; 63.55]$ or $[-3.88\,; 65.32]$

Method 2 may be hard to justify with $n = 10$

Answer B is correct.

We have tested a system for errors and found 0 error in 36 runs. Give a confidence interval for the probability of failure

A. [0% ; 0%]
B. [0%; 1.74%]
C. [0% ; 9.74%]
D. [0% ; 33.74%]
E. [0 ; 100%]
F. I don't know



Solution

CI is $[0\,;p_0(n)]$ with $p_0(n) = 1 - \left(\frac{1-\gamma}{2}\right)^{\frac{1}{n}} \approx \frac{3.689}{n}$

Exact formula gives $p_0(n) = 9.74\%$

Approximation gives $p_0(n) \approx \frac{3.689}{36} = 10.2\%$

Answer C is correct.

We do $n = 100$ experiments and compute a 95%-confidence interval $[L, U]$ for the median. We find $L = 7.4$ and $U = 8.3$. Which statement is correct.

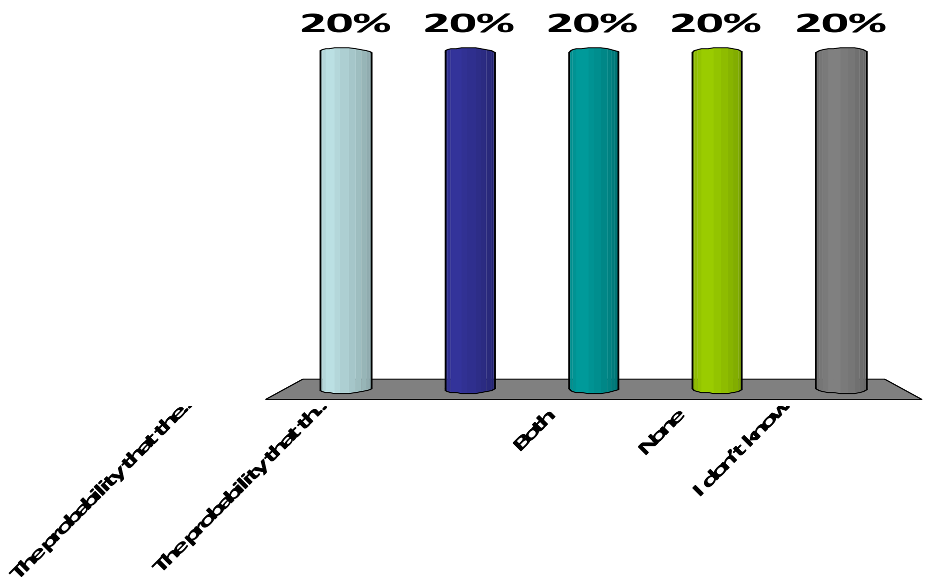
A. The probability that the median is in L ;U is at least 95%

B. The probability that the median is in [7.4 ; 8.3] is at least 95%

C. Both

D. None

E. I don’t know



Solution

A is correct. In other words, we assume that the median is well defined, but hidden to us. The probability that *one* experiment gives a 95%-confidence interval $[L\ ;U]$that contains the median is at least 95%.

B is not correct. In classic statistics, there is no probability assigned to events expressed in terms of the parameter of the distribution.

Answer A.

We expect…

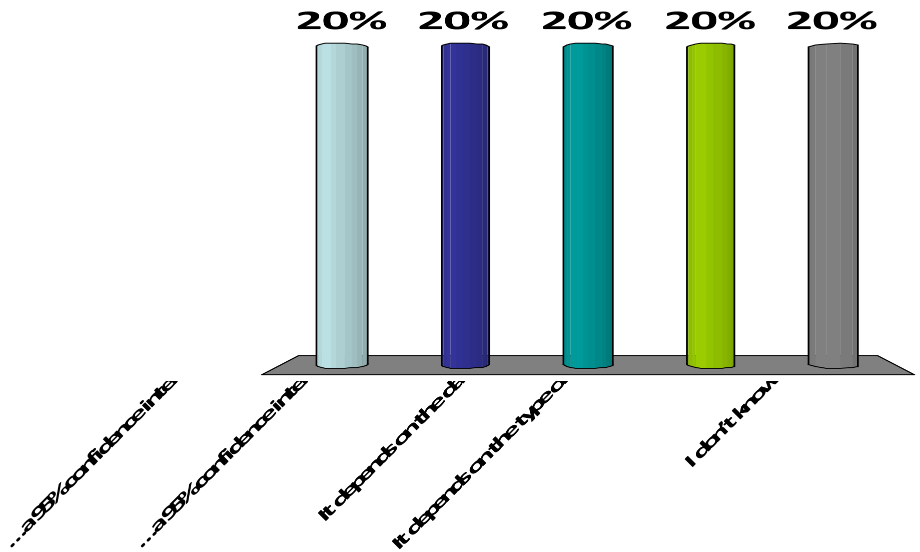
A. … a 95% confidence interval to be narrower than a 99% confidence interval

B. … a 95% confidence interval to be wider than a 99% confidence interval

C. It depends on the data

D. It depends on the type of confidence interval

E. I don't know



Solution

Answer A

If we want to be more certain, we have to enlarge the confidence interval. Check this with the confidence interval for the median $(n = 37)$:

$$95\%: [x_{(13)}; x_{(25)}]$$

$$99\%: [x_{(11)}; x_{(27)}]$$

for the mean:

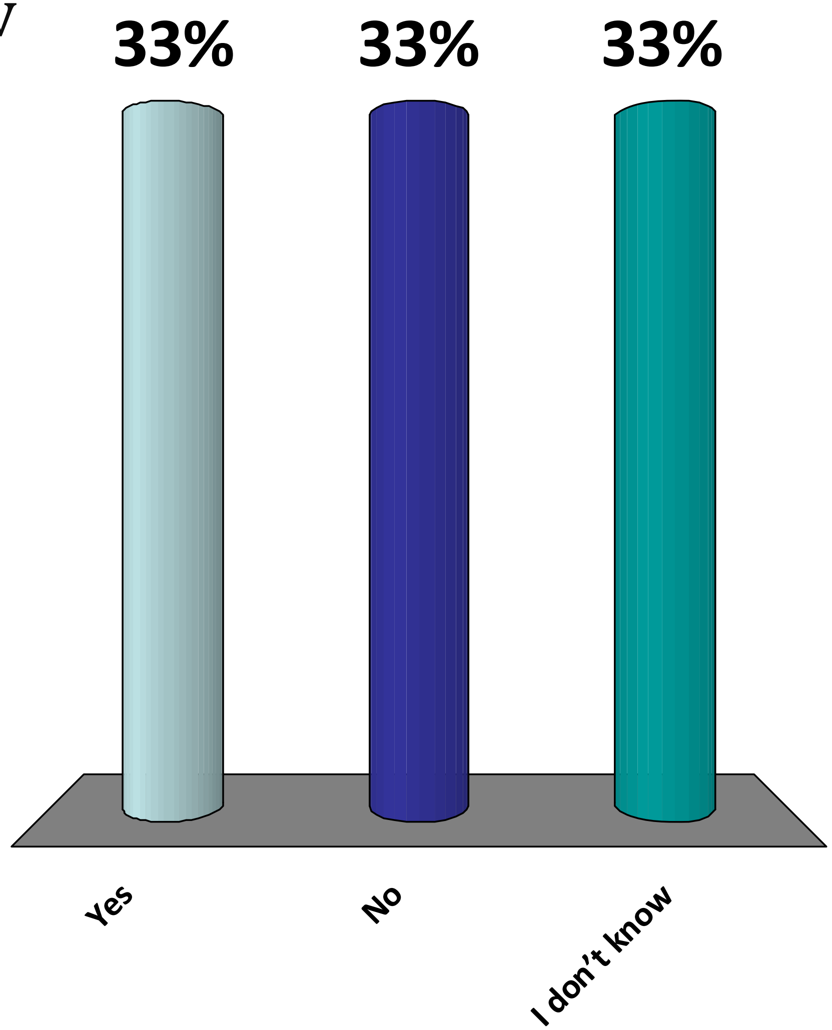
$$95\%: m \pm 1.96 \frac{s}{\sqrt{n}}$$

$$99\%: m \pm 2.58 \frac{s}{\sqrt{n}}$$

A data set $x_i > 0$ is such that $y_i = 1/x_i$ looks normal. A 95% confidence interval for the mean of y_i is $[L; U]$.

Is it true that a confidence interval for the mean of x_i is $\left[\frac{1}{U}; \frac{1}{L}\right]$?

A. Yes
B. No
C. I don't know



Solution

Answer B

$Y_i = 1/X_i$ is a non linear transformation, the mean is not preserved:

$$m_X \neq \frac{1}{m_y}$$

in general.

$1/m_Y$ (i.e. the inverse of the mean of the inverses) is called the *harmonic mean.*

Note that with probability 95%, $L \leq m_Y \leq U$

$[\frac{1}{U}; \frac{1}{L}]$ is a confidence interval for the harmonic mean of x_i

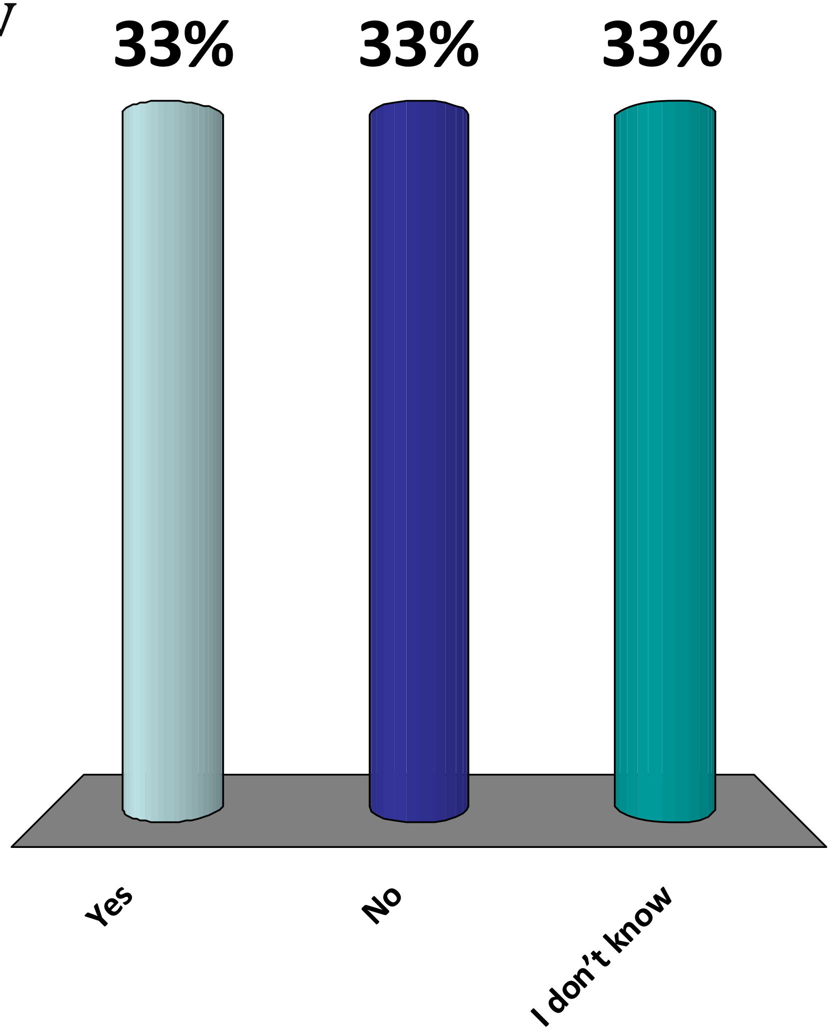
A data set $x_i > 0$ is such that $y_i = 1/x_i$ looks normal. A 95% confidence interval for the median of y_i is $[L\,;U]$.

Is it true that a confidence interval for the median of x_i is $\left[\frac{1}{U};\frac{1}{L}\right]$?

A. Yes

B. No

C. I don't know



Solution

Answer A

$Y_i = 1/X_i$ is a monotonic decreasing transformation, the median is preserved: $\text{median}_X = \frac{1}{\text{median}_Y}$

Further:

$$P(L \leq \text{median}_Y \leq U) \geq 0.95$$

$$(L \leq \text{median}_X \leq U) \Leftrightarrow \left(\frac{1}{U} \leq \frac{1}{\text{median}_Y} \leq \frac{1}{L}\right)$$

$$\Leftrightarrow (\frac{1}{U} \leq \text{median}_X \leq \frac{1}{L})$$

Thus $P\left(\frac{1}{U} \leq \text{median}_X \leq \frac{1}{L}\right) \geq 0.95$