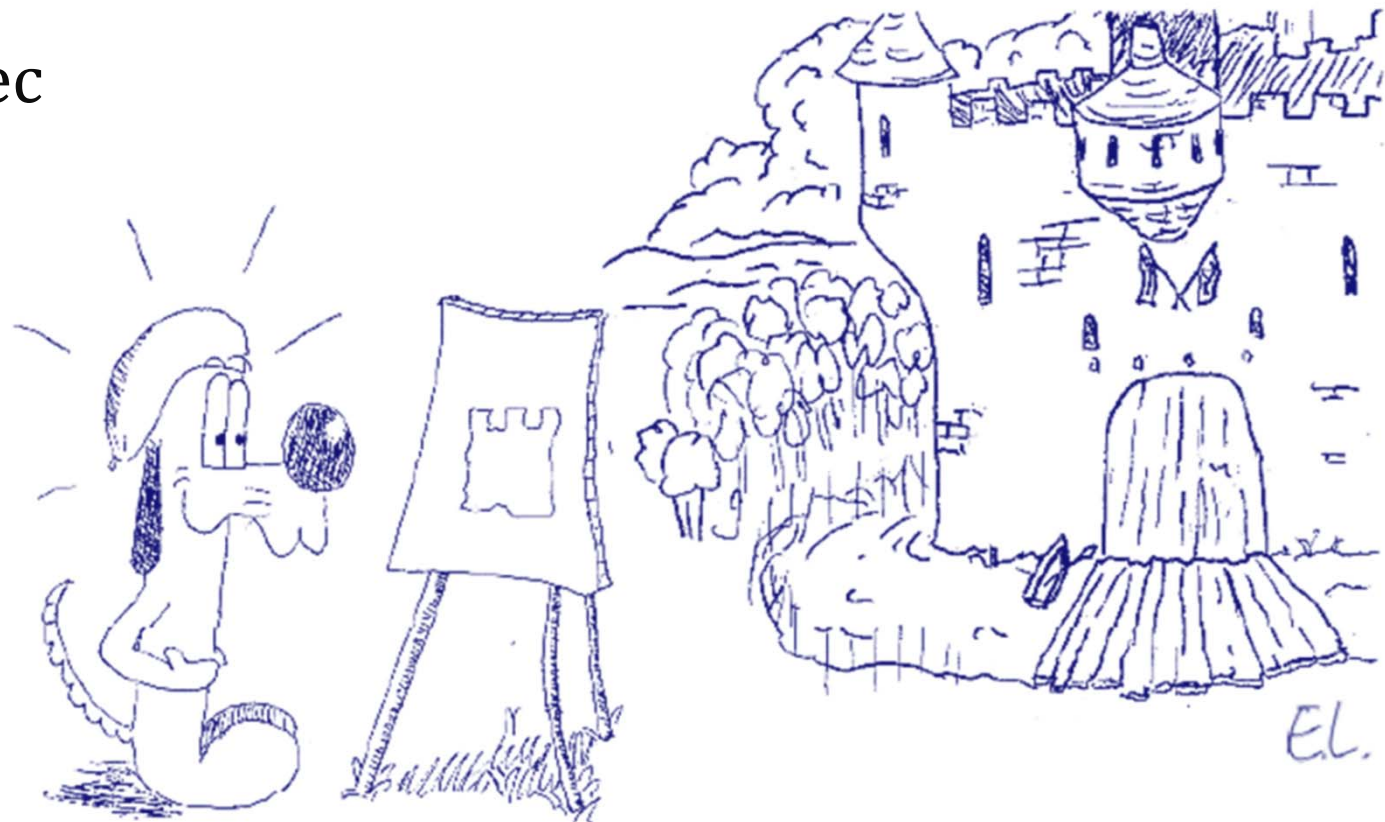


Model Fitting Bonus

JY Le Boudec

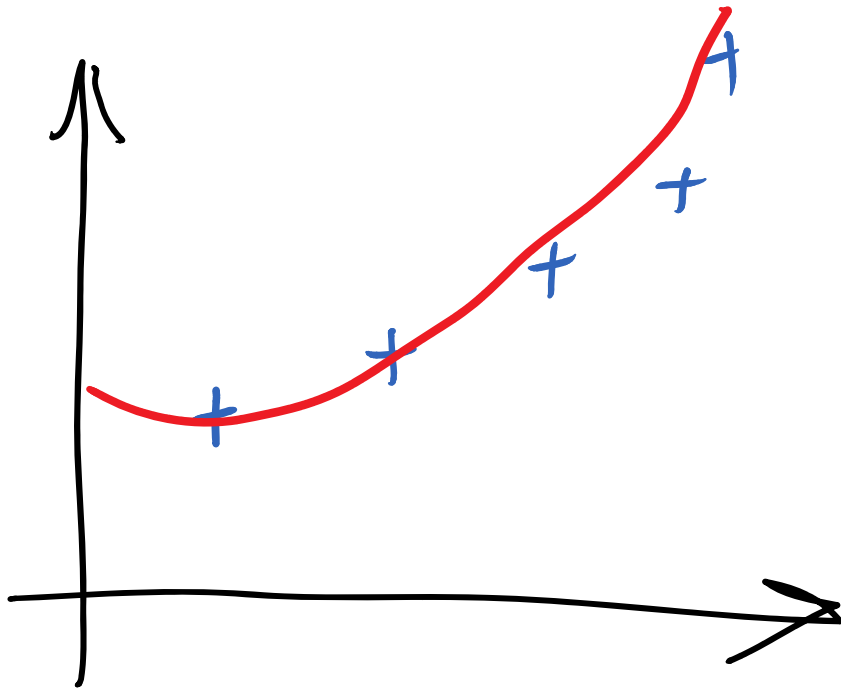
May 2015



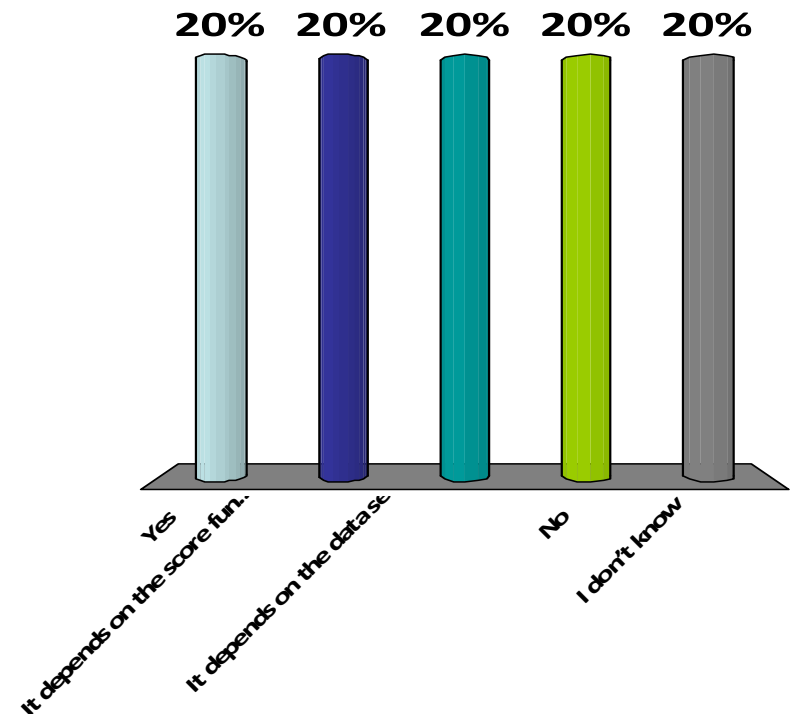
We want to fit a data set y_i to a polynomial of degree 2:

$$y_i = at_i^2 + bt_i + c.$$

Is this a linear regression model ?

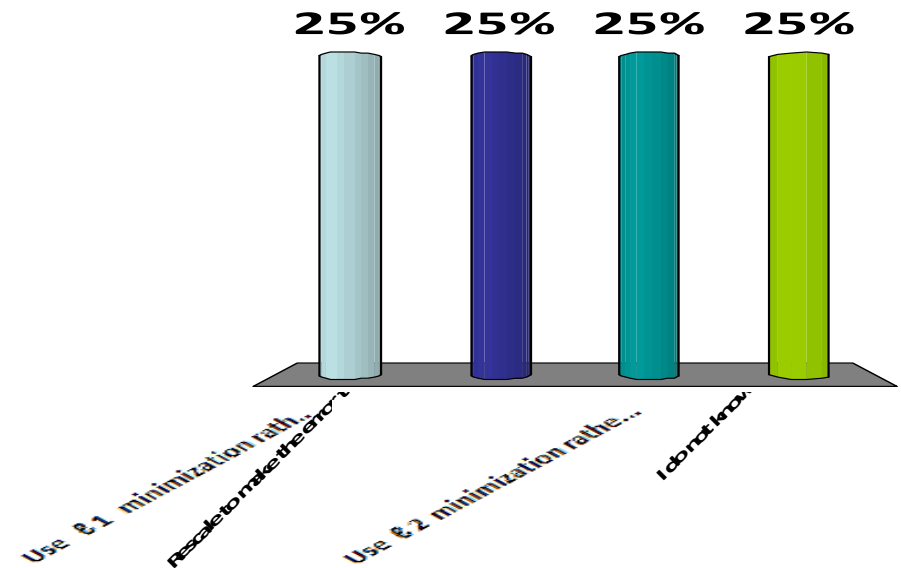


- A. Yes
- B. It depends on the score function
- C. It depends on the data set
- D. No
- E. I don't know



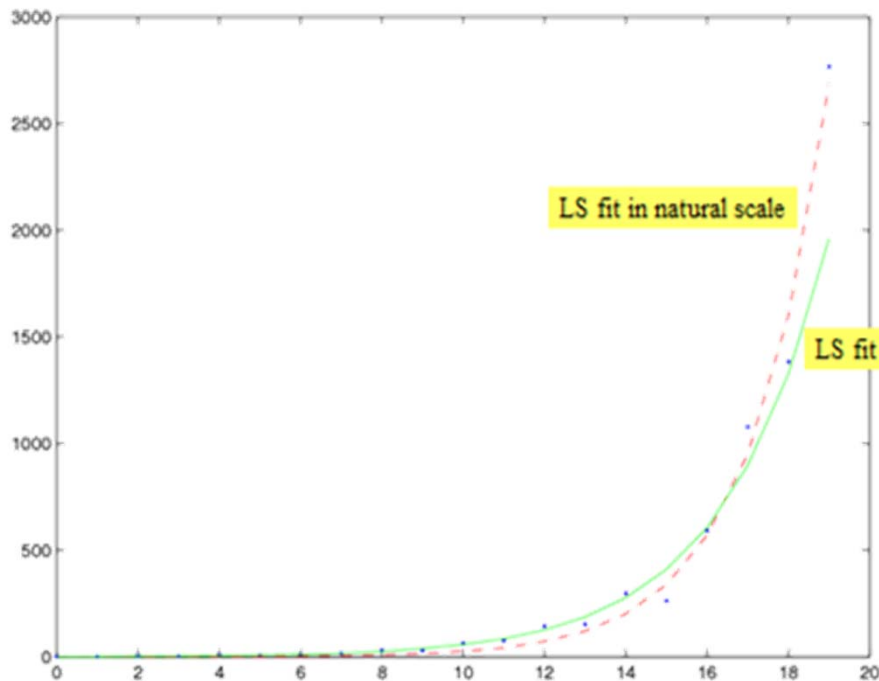
If the error terms in a fitting model are not homoscedastic, it is better to ...

- A. Use ℓ^1 minimization rather than ℓ^2
- B. Rescale to make the error term homoscedastic
- C. Use ℓ^2 minimization rather than ℓ^1
- D. I do not know

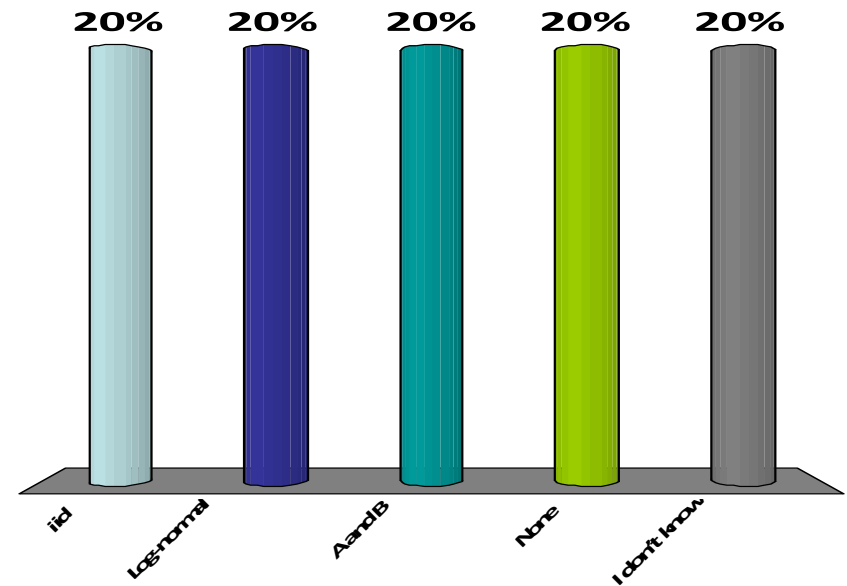


The green estimation corresponds to assuming that the error terms (blue dot – green curve) are ...

- A. iid
- B. Log-normal
- C. A and B
- D. None
- E. I don't know



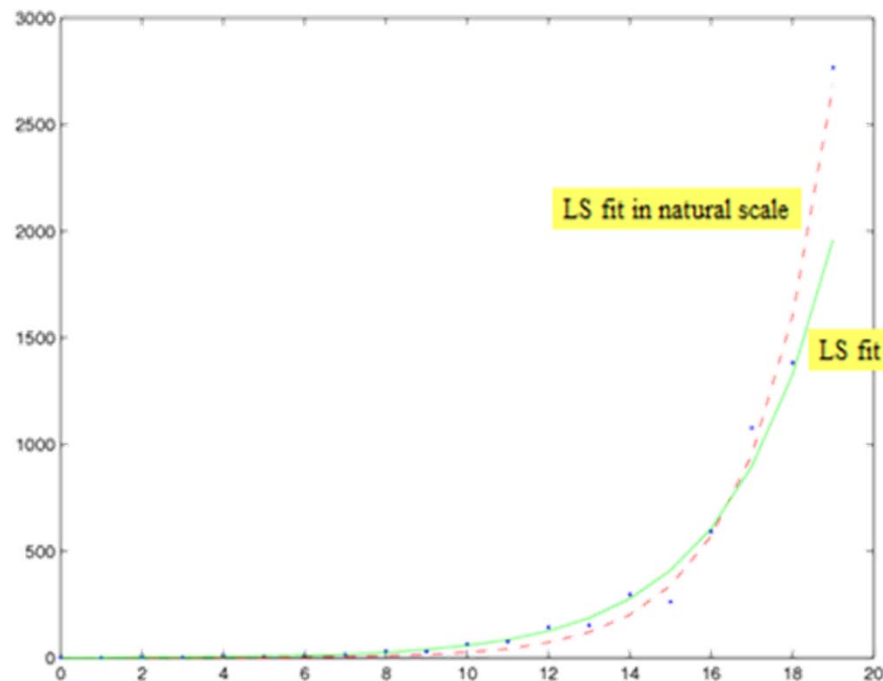
5



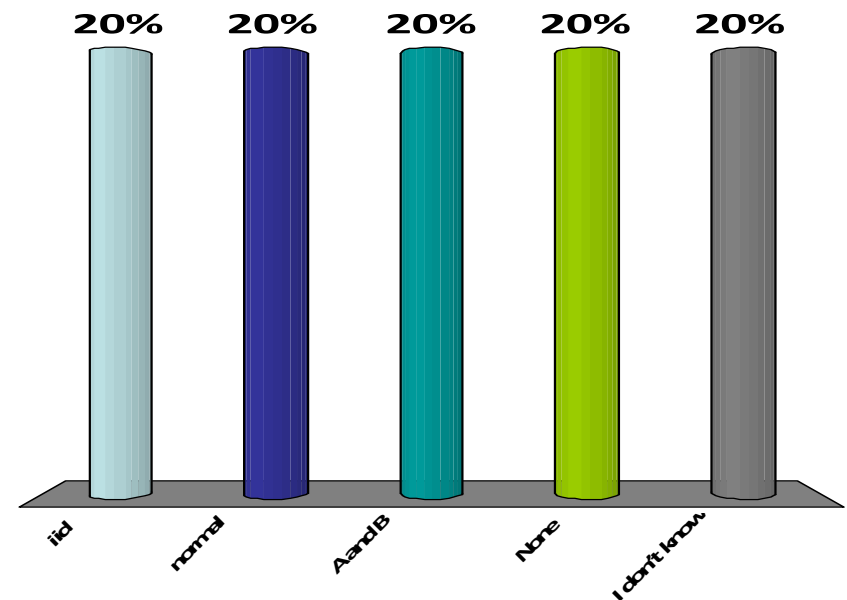
6

The green estimation corresponds to assuming that the *relative* error terms (blue dot – green curve) are ...

- A. iid
- B. normal
- C. A and B
- D. None
- E. I don't know



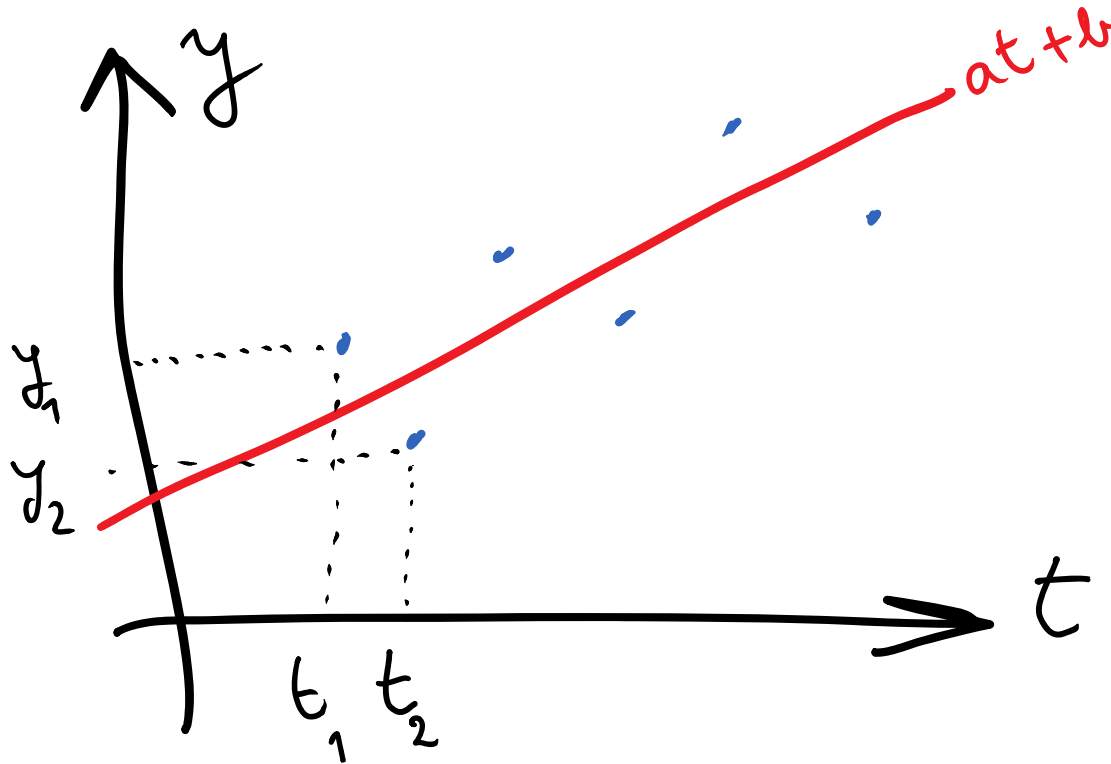
5



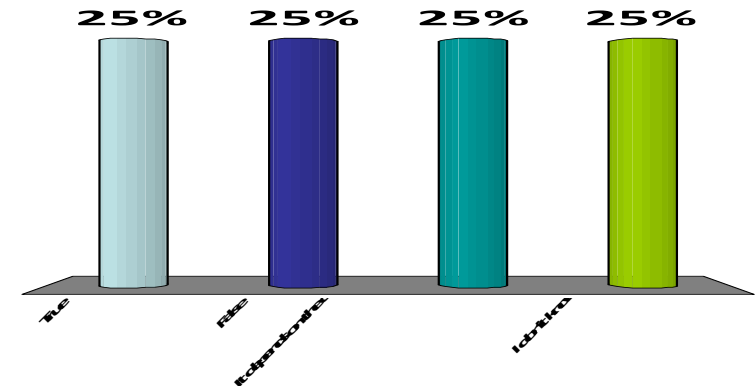
8

We fit the model $y_i = at_i + b$ using least squares.

The obtained line is such that the average distance from the points to the line is 0.

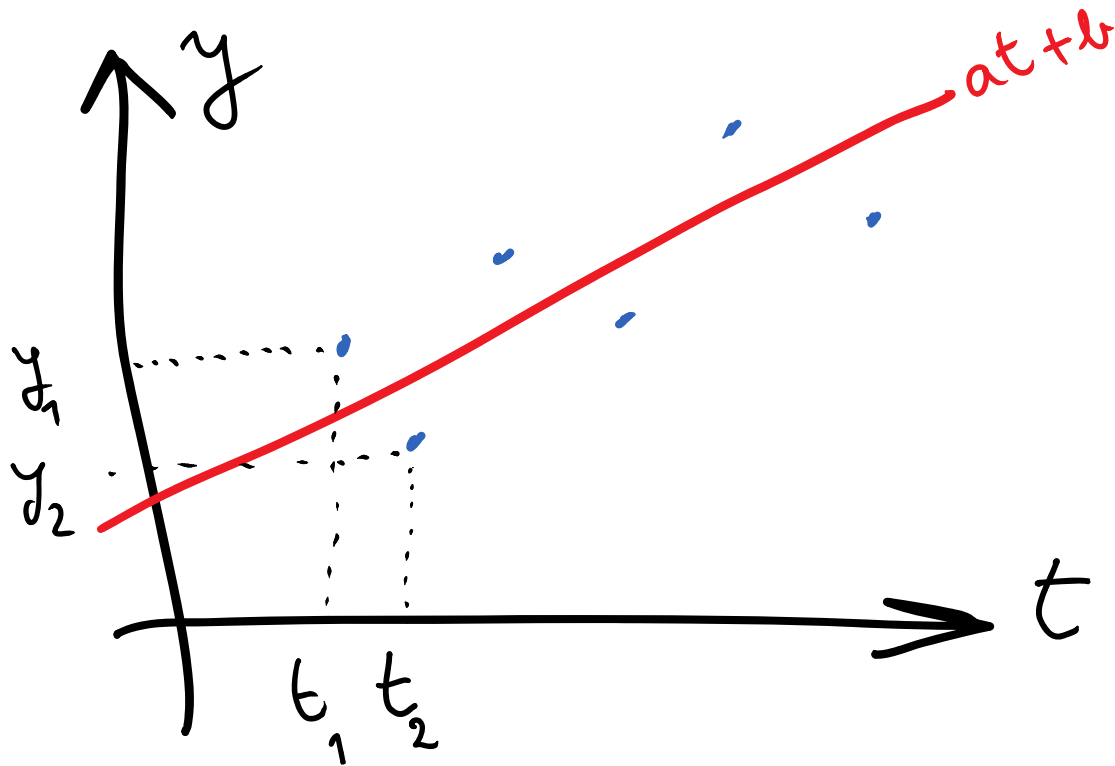


- A. True
- B. False
- C. It depends on the data
- D. I don't know

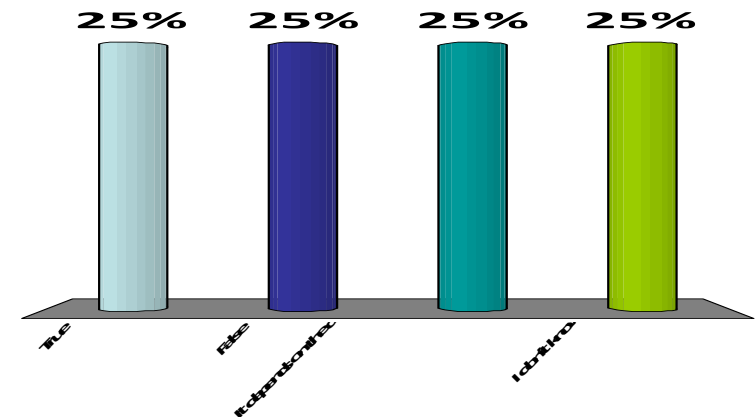


We fit the model $y_i = at_i + b$ using ℓ^1 norm minimization.

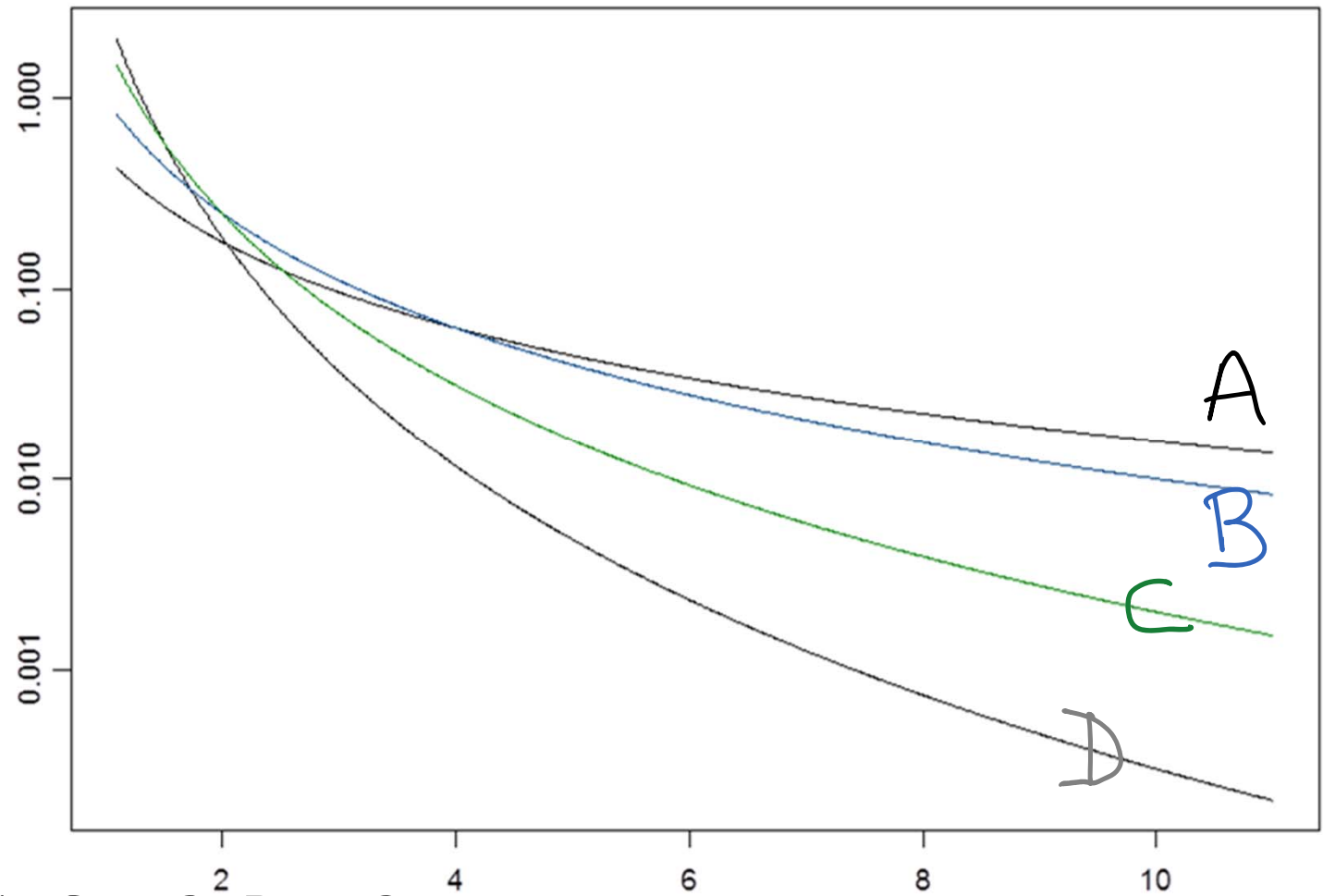
The obtained line leaves an equal number of points on each side



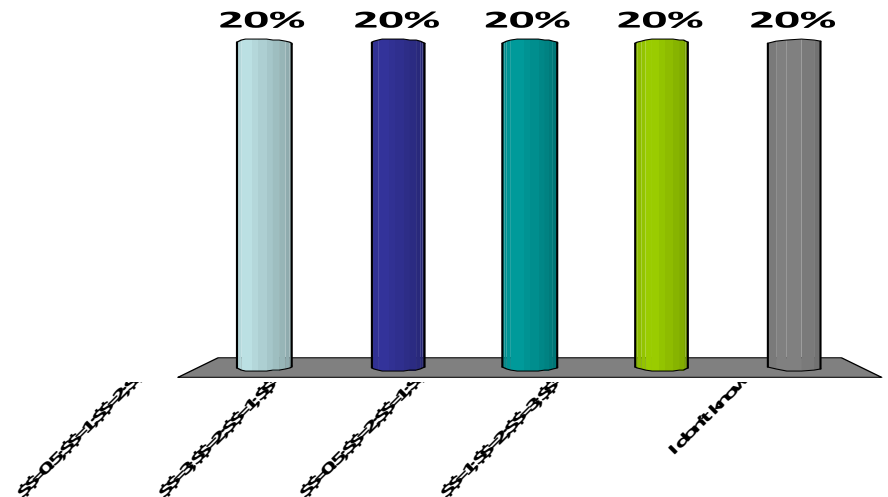
- A. True
- B. False
- C. It depends on the data
- D. I don't know



Find the parameter p for each of these standard Pareto PDFs $f(x)$

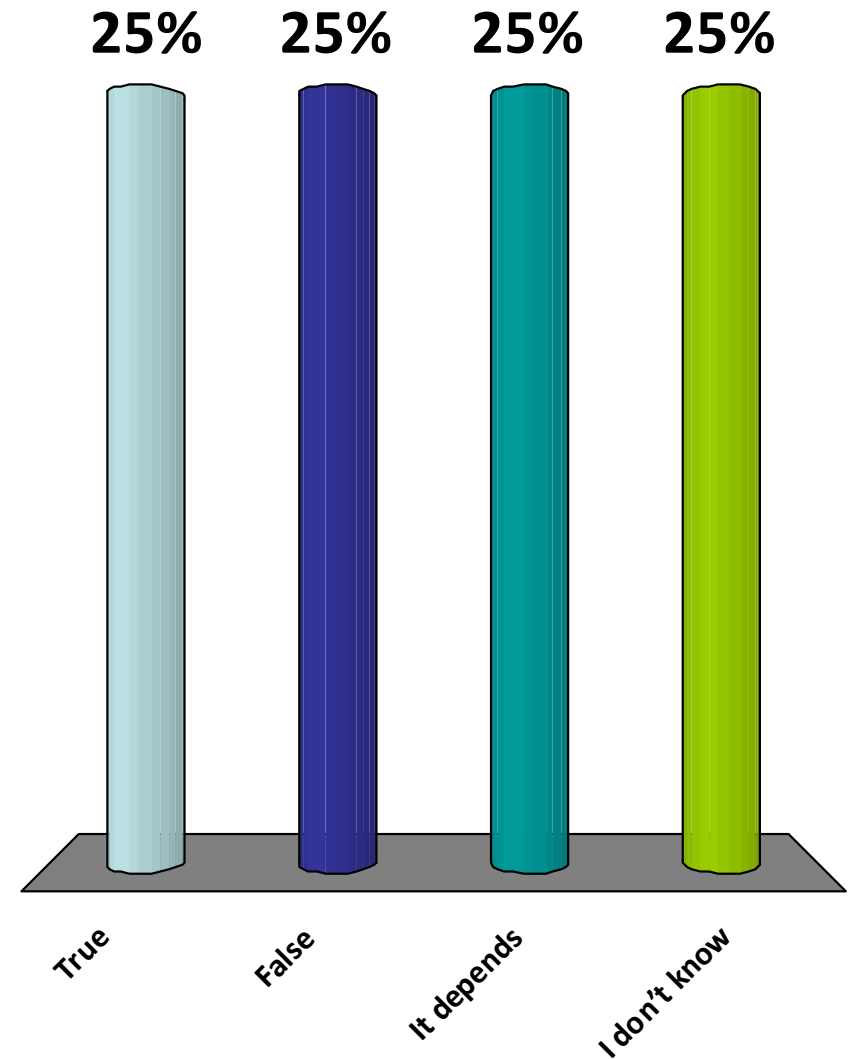
$$= \frac{p}{x^{p+1}} \mathbf{1}_{x>1}$$


- A. $A = 0.5; B = 1; C = 2; D = 3$
- B. $A = 3; B = 2; C = 1; D = 0.5$
- C. $A = 0.5; B = 2; C = 1; D = 3$
- D. $A = 1; B = 2; C = 3; D = 0.5$
- E. I don't know



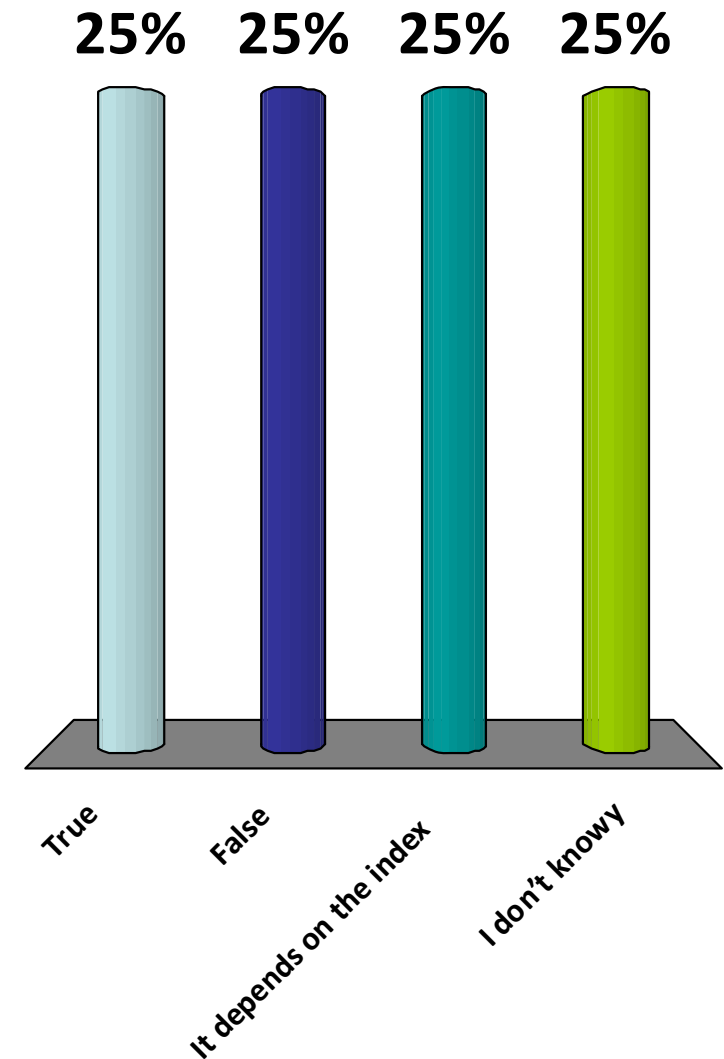
If a positive random variable has a finite mean and is heavy tailed, its variance is infinite

- A. True
- B. False
- C. It depends
- D. I don't know



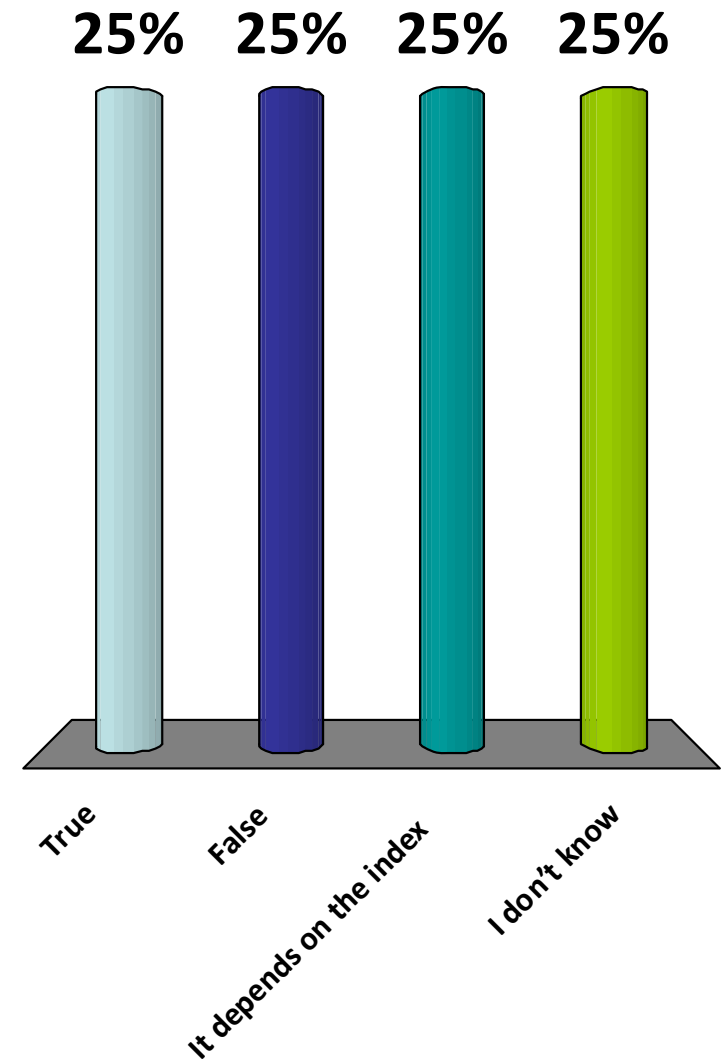
The Complementary CDF of a Pareto distribution follows a power law...

- A. True
- B. False
- C. It depends on the index p
- D. I don't knowy



A Pareto distribution is heavy tailed ...

- A. True
- B. False
- C. It depends on the index p
- D. I don't know

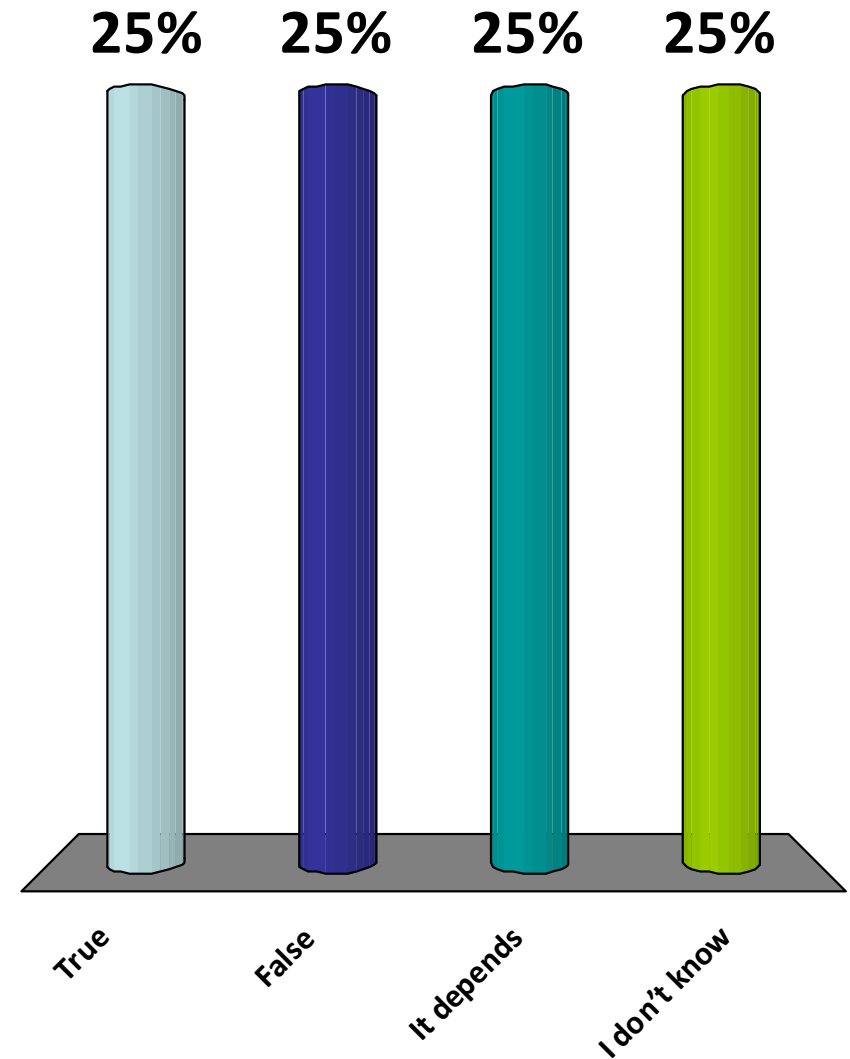


For a Pareto distribution, the hazard rate

$\lambda(t)$ is such that

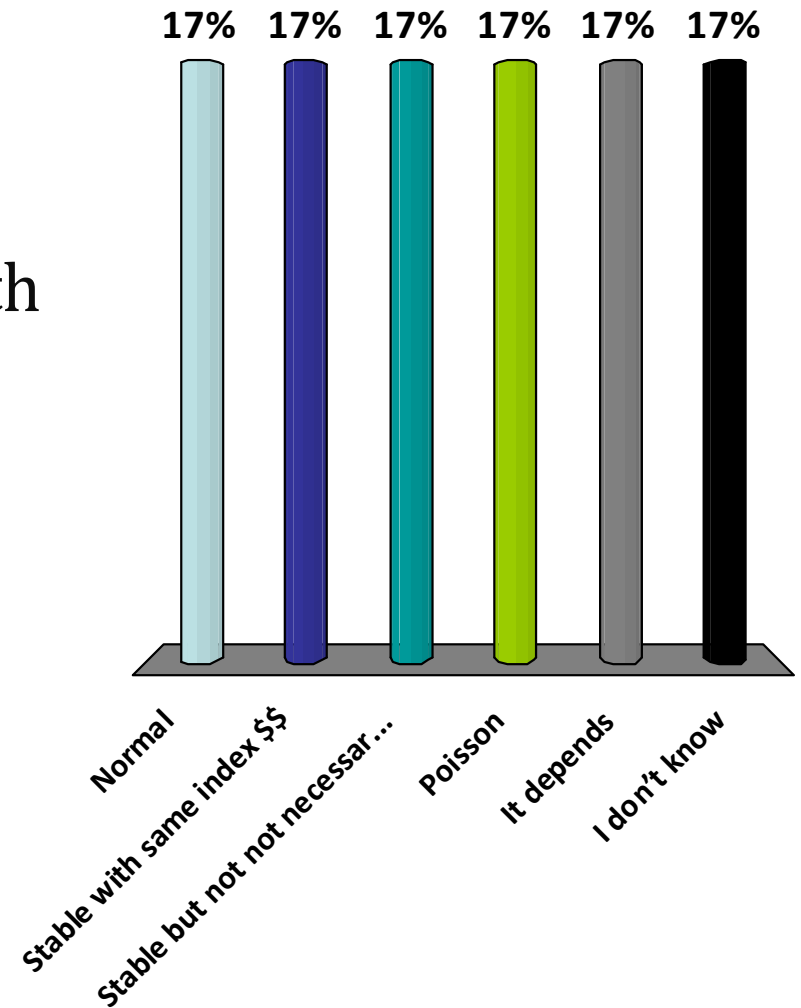
$$\lim_{t \rightarrow \infty} \lambda(t) = 0$$

- A. True
- B. False
- C. It depends
- D. I don't know



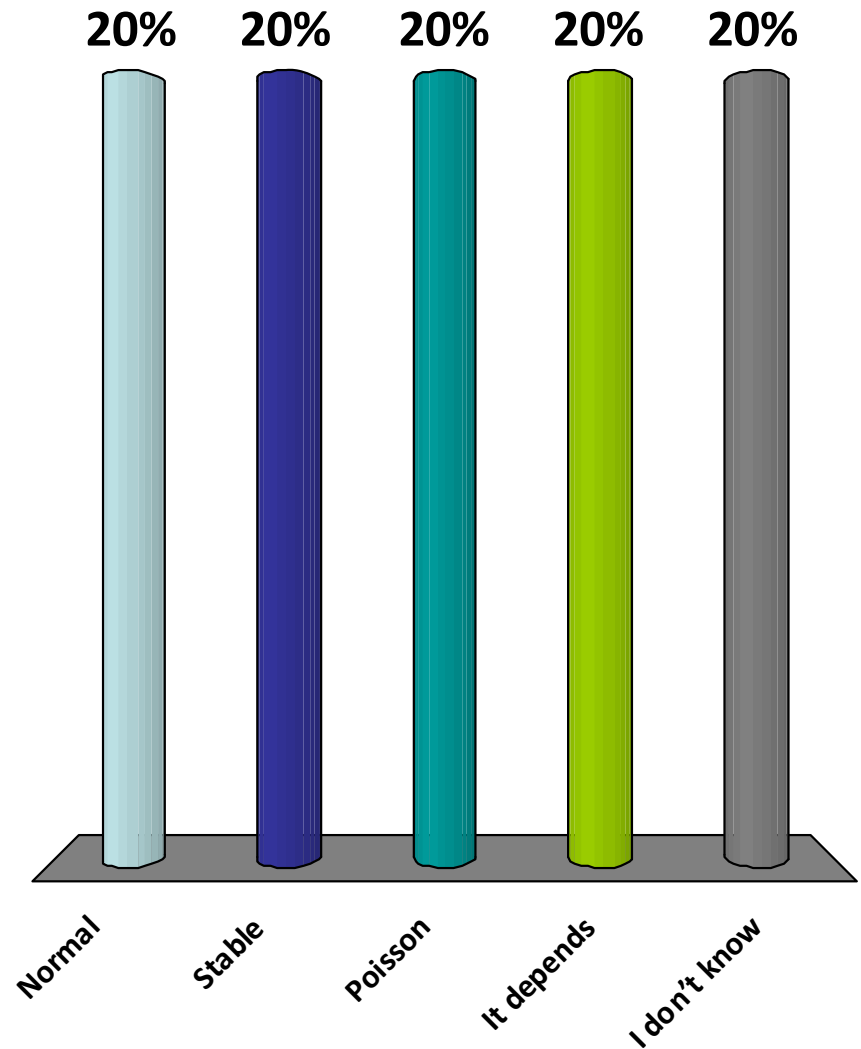
The distribution of the sum of n iid random variables with heavy tail and index $p < 2$, for large n , is approximately...

- A. Normal
- B. Stable with same index p
- C. Stable but not necessarily with same index p
- D. Poisson
- E. It depends
- F. I don't know



The distribution of the sum of n iid random variables with finite variance, for large n , is approximately...

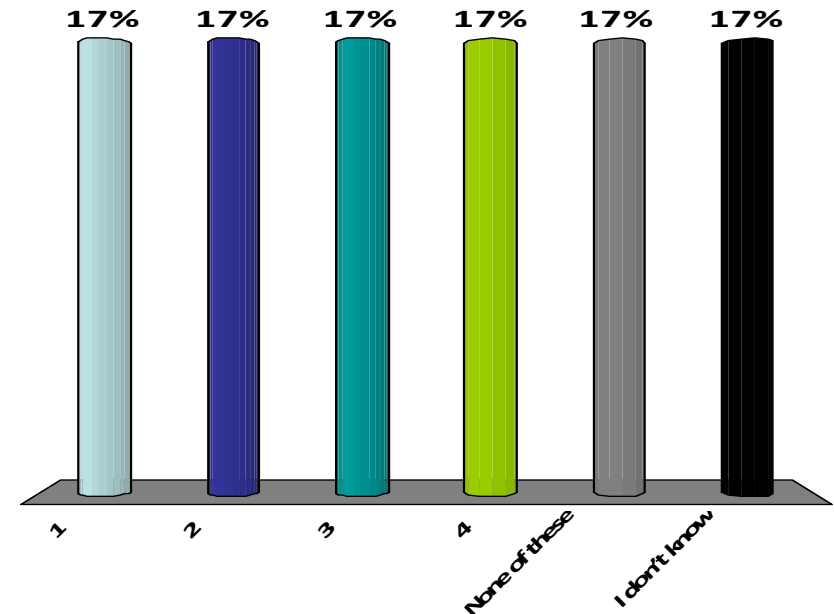
- A. Normal
- B. Stable
- C. Poisson
- D. It depends
- E. I don't know



We want to estimate some quantity μ . We have $m + n$ independent measurements $X_1, \dots, X_m \sim iid N(\mu, \sigma^2)$ and $Y_1, \dots, Y_n \sim iid N(\mu, \lambda^2 \sigma^2)$. λ is known. We do weighted least square estimation of μ . What do we obtain ?

1. $\hat{\mu}_1 = \frac{X_1 + \dots + X_m + Y_1 + \dots + Y_n}{m+n}$
2. $\hat{\mu}_2 = \frac{X_1 + \dots + X_m + \lambda(Y_1 + \dots + Y_n)}{m + \lambda n}$
3. $\hat{\mu}_3 = \frac{X_1 + \dots + X_m + \frac{Y_1 + \dots + Y_n}{\lambda}}{m + \frac{n}{\lambda}}$
4. $\hat{\mu}_4 = \frac{X_1 + \dots + X_m + \frac{Y_1 + \dots + Y_n}{\lambda^2}}{m + \frac{n}{\lambda^2}}$

- A. 1
- B. 2
- C. 3
- D. 4
- E. None of these
- F. I don't know



We want to estimate some quantity μ . We have $m + n$ independent measurements

$$X_1, \dots, X_m \sim iid N(\mu, \sigma^2) \text{ and } Y_1, \dots, Y_n \sim iid N(\mu, \lambda^2 \sigma^2)$$

σ and λ are unknown but we think that $\lambda \gg 1$. i.e. Y is very noisy; $m \approx n$

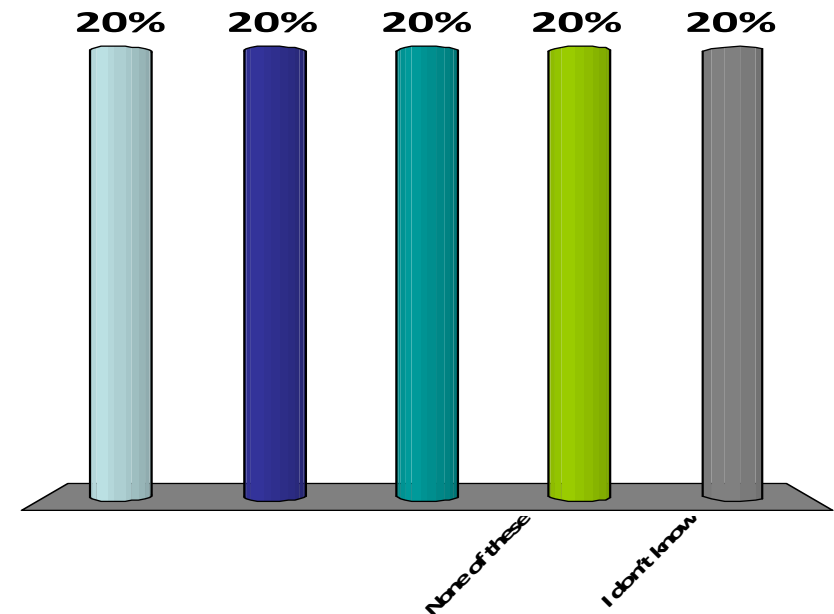
To estimate μ , say which formula you prefer:

1. $\hat{\mu}_1 = \frac{X_1 + \dots + X_m}{m}$

2. $\hat{\mu}_2 = \frac{Y_1 + \dots + Y_n}{n}$

3. $\hat{\mu}_3 = \frac{X_1 + \dots + X_m + Y_1 + \dots + Y_n}{m+n}$

- 1.
- 2.
- 3.
- None of these
- I don't know



We want to estimate some quantity μ . We have $m + n$ independent measurements

$$X_1, \dots, X_m \sim iid N(\mu, \sigma^2) \text{ and } Y_1, \dots, Y_n \sim iid N(\mu, \lambda^2 \sigma^2)$$

σ and λ are unknown but we think that $\lambda \gg 1$.

- A. 1
- B. 2
- C. 3
- D. 4
- E. None of these
- F. I don't know

To estimate μ , say which formula you prefer:

$$1. \hat{\mu}_1 = \frac{X_1 + \dots + X_m}{m}$$

$$2. \hat{\mu}_2 = \frac{Y_1 + \dots + Y_n}{n}$$

$$3. \hat{\mu}_3 = \frac{X_1 + \dots + X_m + Y_1 + \dots + Y_n}{m+n}$$

$$4. \hat{\mu}_4 = \frac{X_1 + \dots + X_m + \frac{Y_1 + \dots + Y_n}{\lambda^2}}{m + \frac{n}{\lambda^2}}$$

