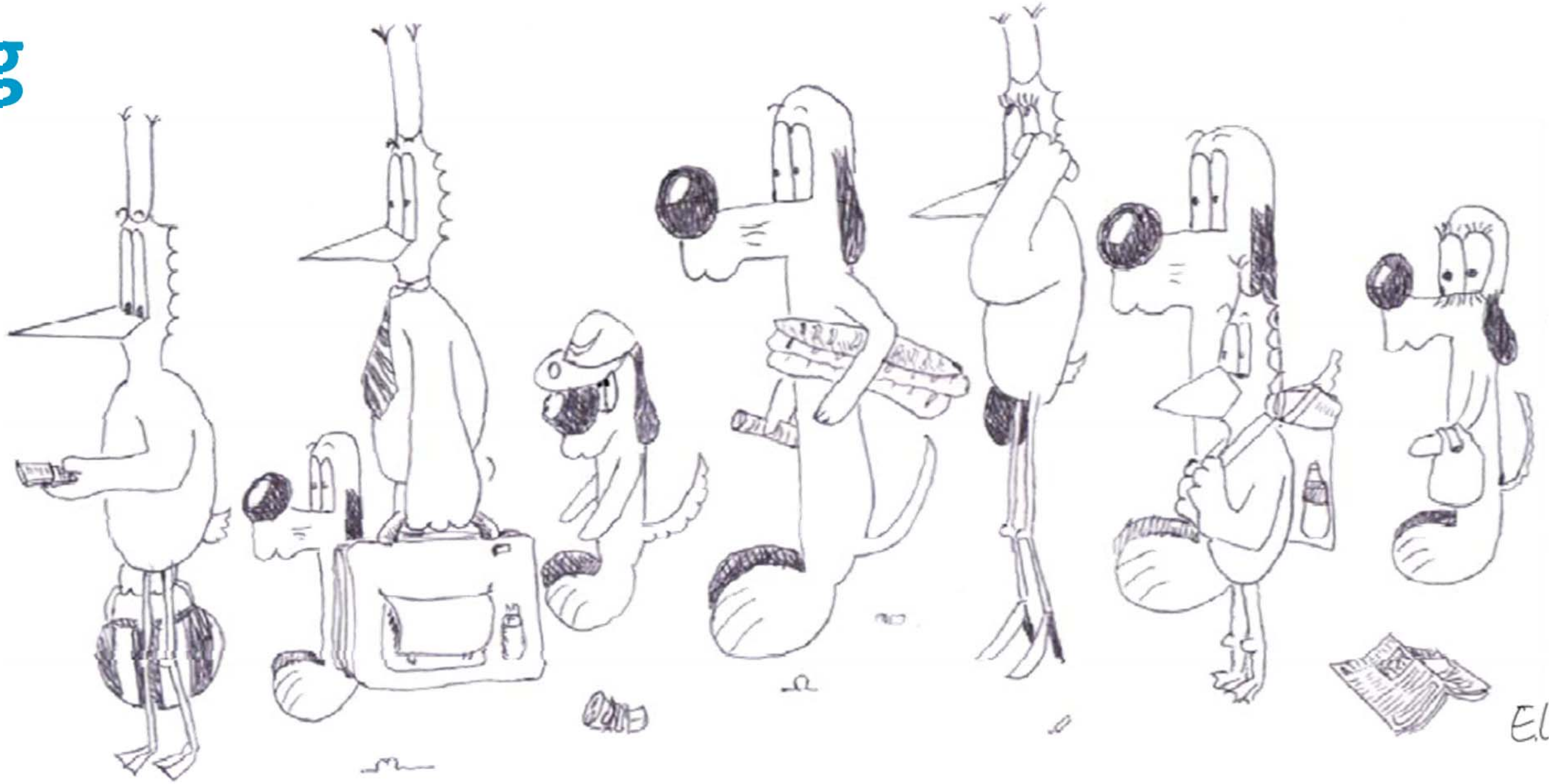
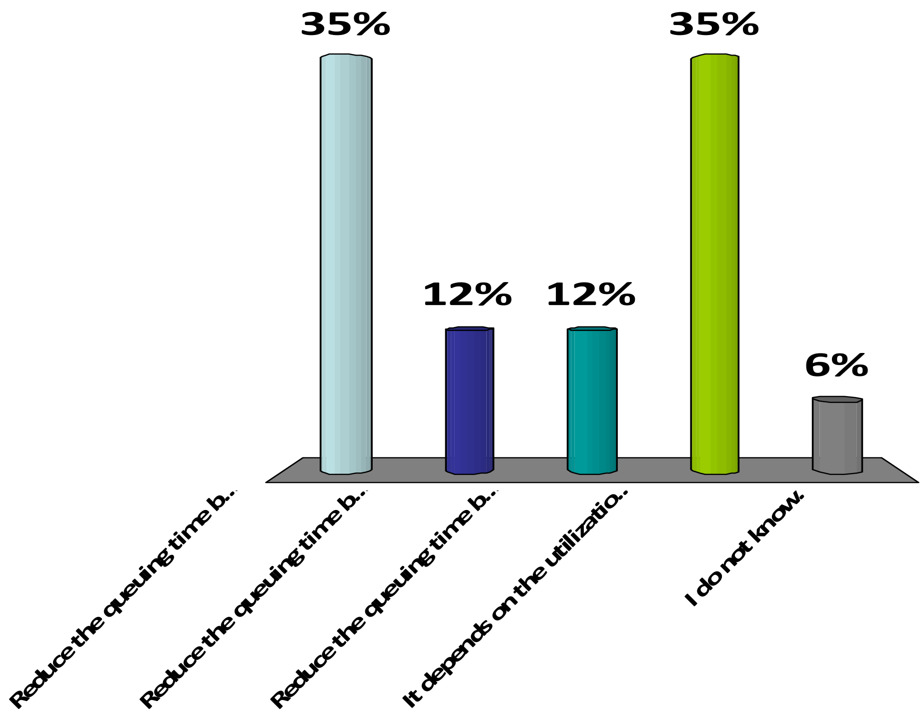
Bonus
Queuing



Jean-Yves Le Boudec
2015

An information server can be modelled as an M/GI/1 queue. Doubling the capacity of the server would…

A. Reduce the queuing time by a factor 2
B. Reduce the queuing time by a factor larger than 2
C. Reduce the queuing time by a factor smaller than 2
D. It depends on the utilization factor
E. I do not know.



Solution

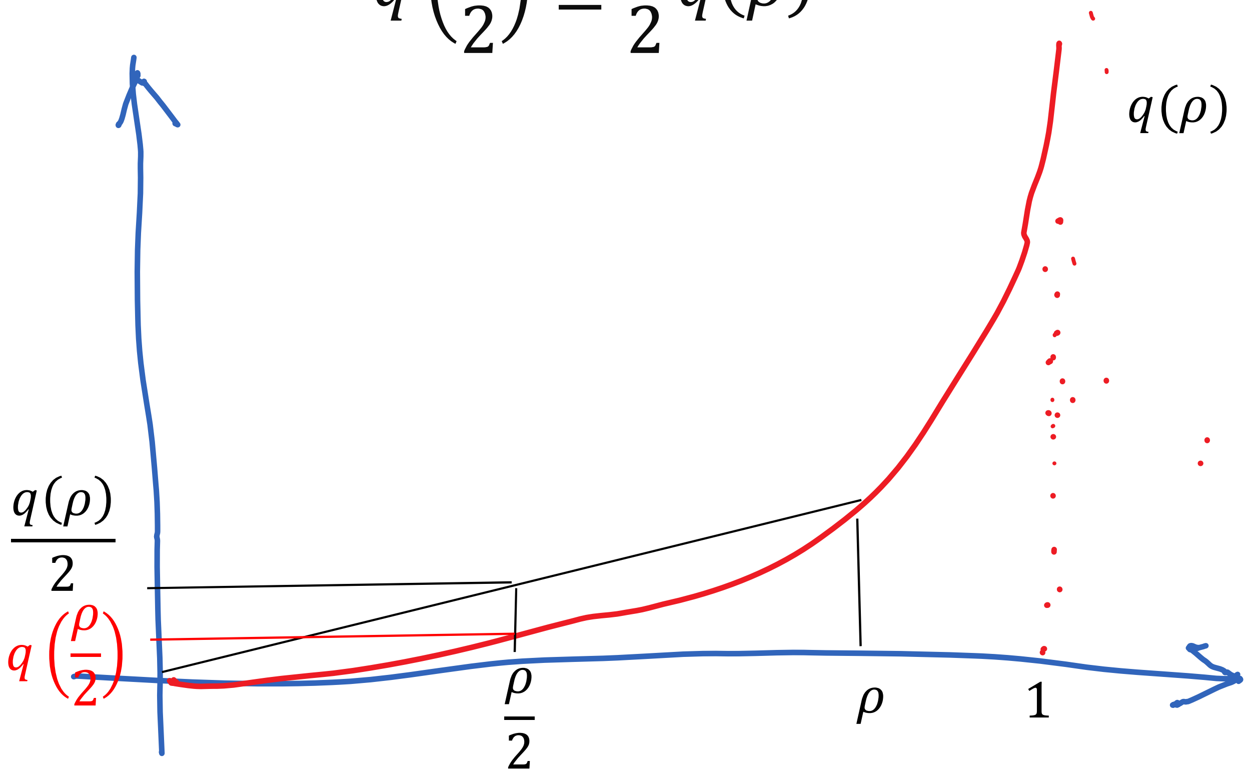
Answer B

Doubling the capacity $\Rightarrow \rho$ is divided by 2

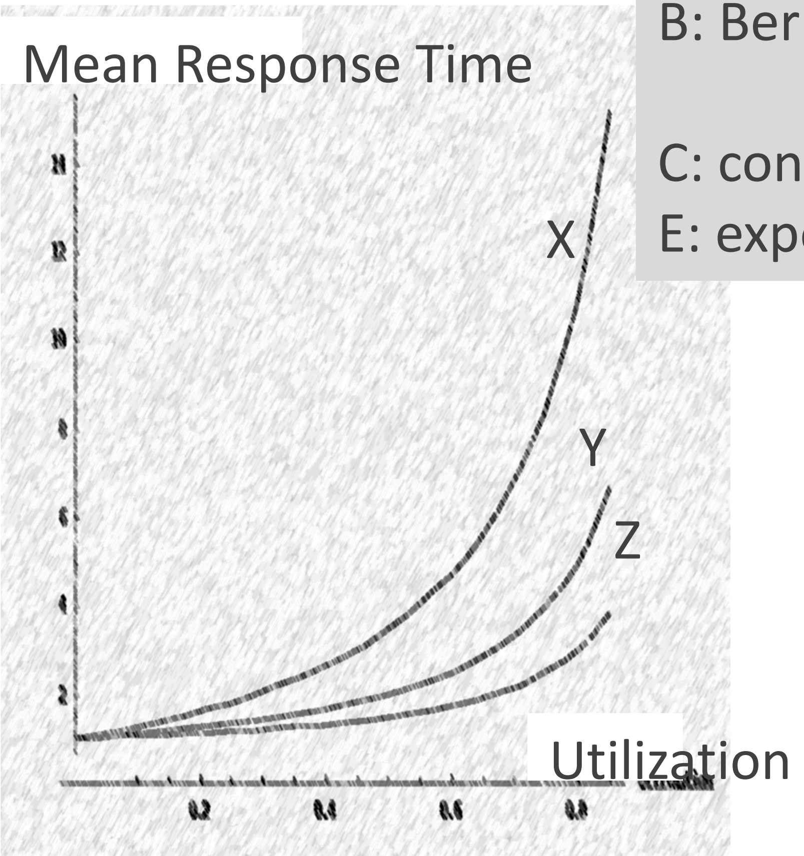
Queuing delay $q(\rho)$ is convex with ρ and $q(0) = 0$

$$q\left(\frac{\rho}{2}\right) \leq \frac{1}{2}\left(q(0) + q(\rho)\right)$$

$$q\left(\frac{\rho}{2}\right) \leq \frac{1}{2} q(\rho)$$



The 3 curves are for an M/GI/1 queue with different distributions of service times. Say which curve is for which distribution.



B: Bernoulli with mean $p = 0.2$

C: constant

E: exponential

A. X=B, Y=C; Z=E

B. X=B, Y=E; Z=C

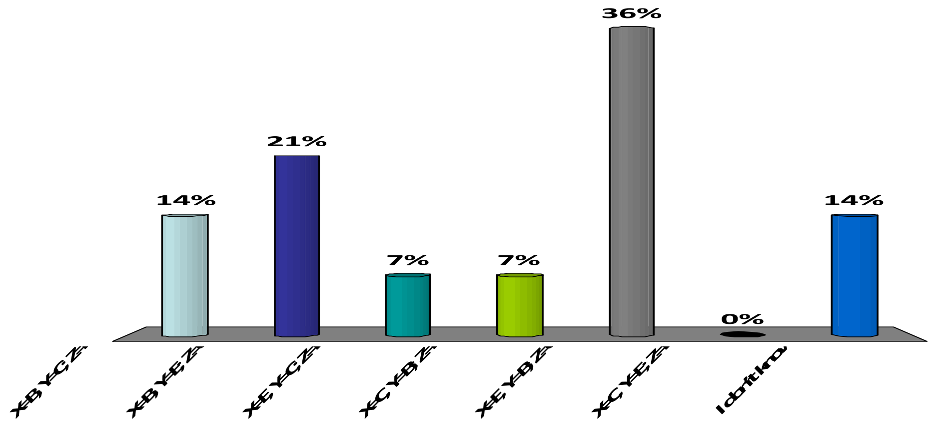
C. X=E, Y=C; Z=B

D. X=C, Y=B; Z=E

E. X=E, Y=B; Z=C

F. X=C, Y=E; Z=B

G. I don't know



Solution

The curves are ranked by the CoV of the distributions.
Exponential: CoV = 1

Constant: CoV =0

Bernoulli:
mean = 0.2
variance = $p(1-p) = 0.2 \times 0.8 = 0.16$
standard deviation = 0.4
CoV = $\frac{0.4}{0.2} = 2$

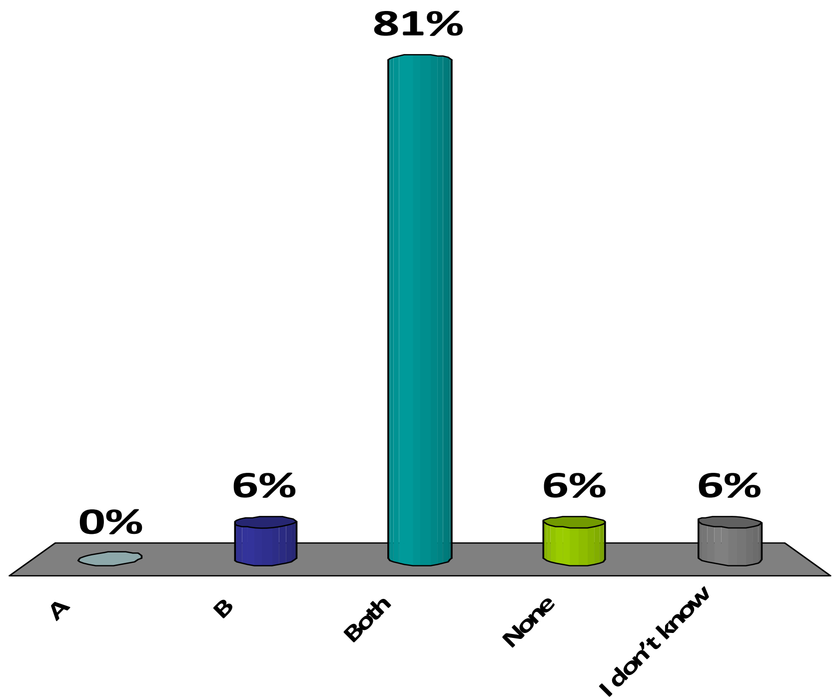
Answer B

Which sentences are true ?
$\lambda =$ arrival rate
$S =$ mean service time

A. For a single server queue, if $\lambda < \frac{1}{S}$ the queue has a stationary regime

B. For an M/GI/1 queue, if $\lambda < \frac{1}{S}$ the queue has a stationary regime

A. A
B. B
C. Both
D. None
E. I don’t know

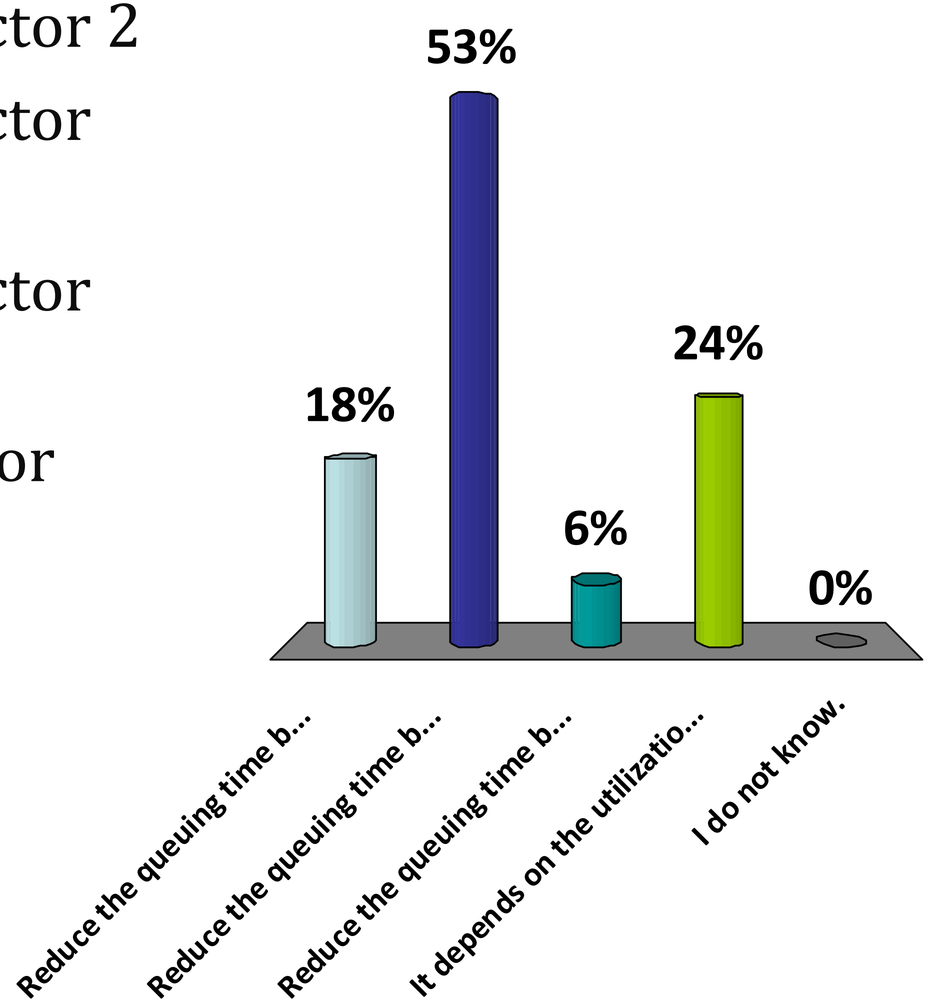


Solution

Loyne' theorem $\Rightarrow$ Answer C

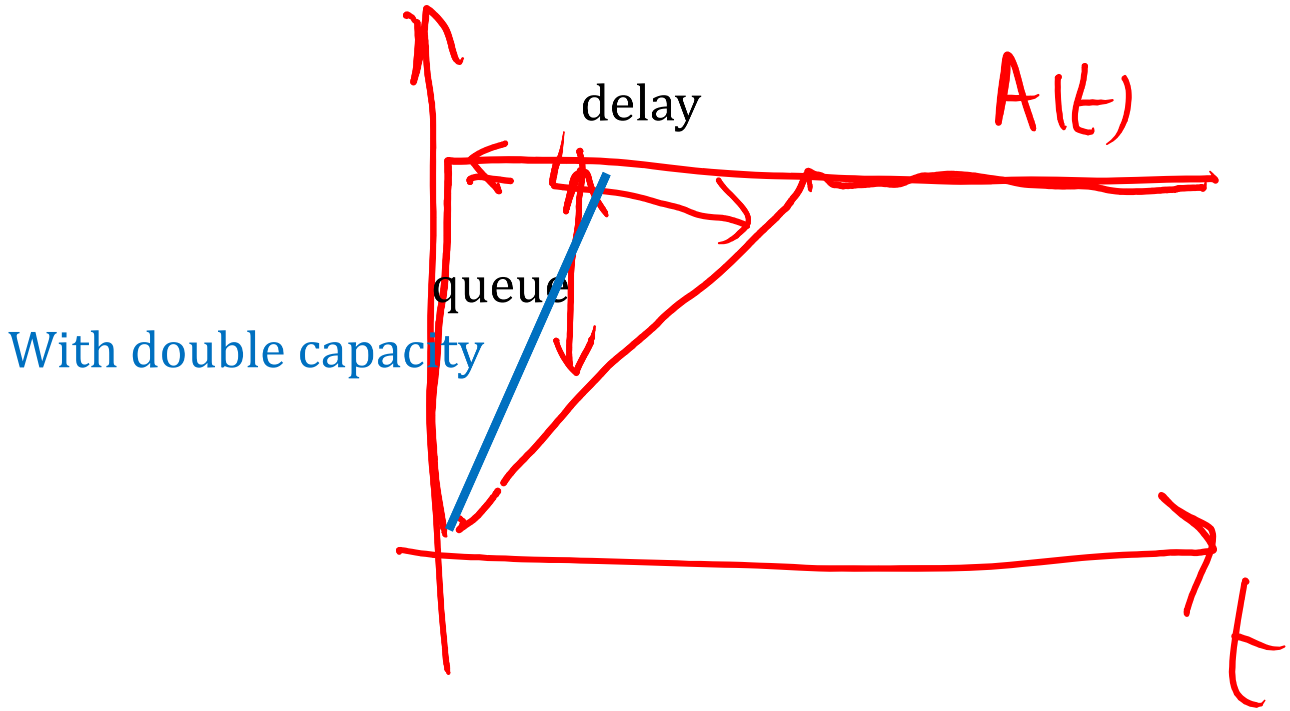
A train with 200 tourists arrive at the skilift. A queue builds up. Doubling the capacity of the skilift would...

A. Reduce the queuing time by a factor 2
B. Reduce the queuing time by a factor larger than 2
C. Reduce the queuing time by a factor smaller than 2
D. It depends on the utilization factor
E. I do not know.



Solution

Answer A



The average number of customers present in an M/GI/∞ queue is … (S is the mean service time)

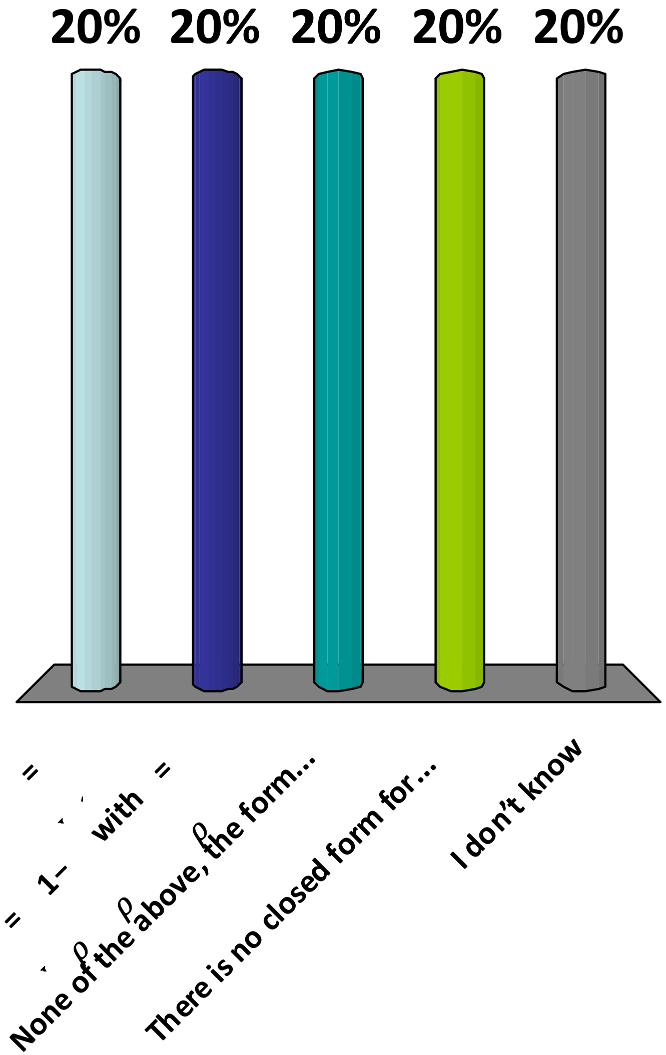
A. $N = \lambda S$

B. $N = \frac{\rho}{1-\rho}$ with $\rho = \lambda S$

C. None of the above, the formula involves the coefficient of variation

D. There is no closed form formula

E. I don’t know



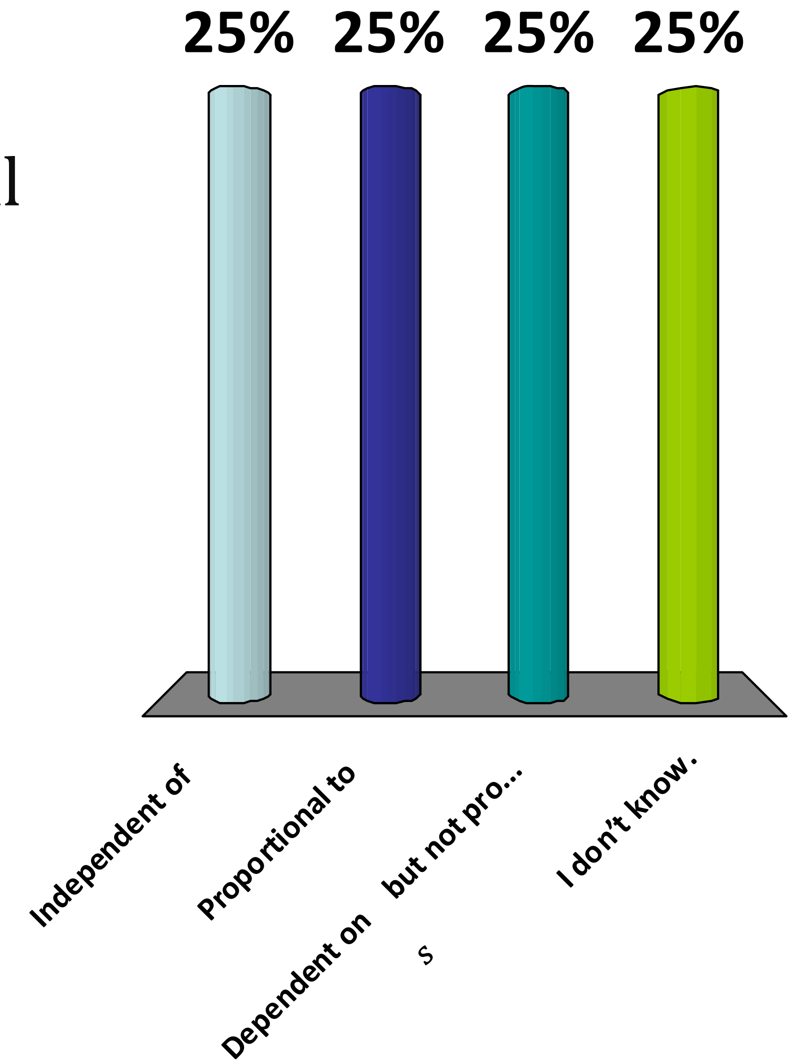
Solution

The response time is the service time

Little's theorem ⇒   Answer A

At a FIFO queue, the expected waiting time for a job, given that its service time is *s* is…

A. Independent of *s*
B. Proportional to *s*
C. Dependent on *s* but not proportional (in general)
D. I don’t know.



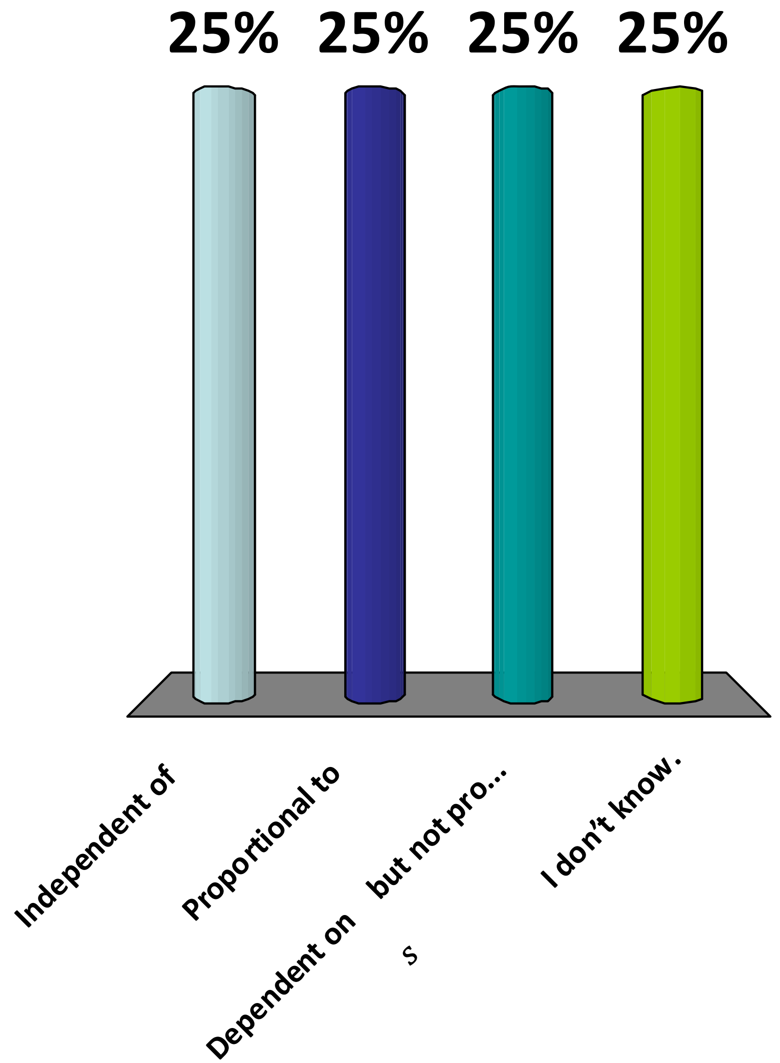
Solution

The waiting time depends only on the amount of unfinished work found upon arrival (FIFO property)

$\Rightarrow$ Answer A

At a PS (processor sharing) queue, the expected response time for a job, given that its service time is *s* is…

A. Independent of *s*
B. Proportional to *s*
C. Dependent on *s* but not proportional (in general)
D. I don't know.



Solution

Fair sharing property of PS ⇒ Answer B

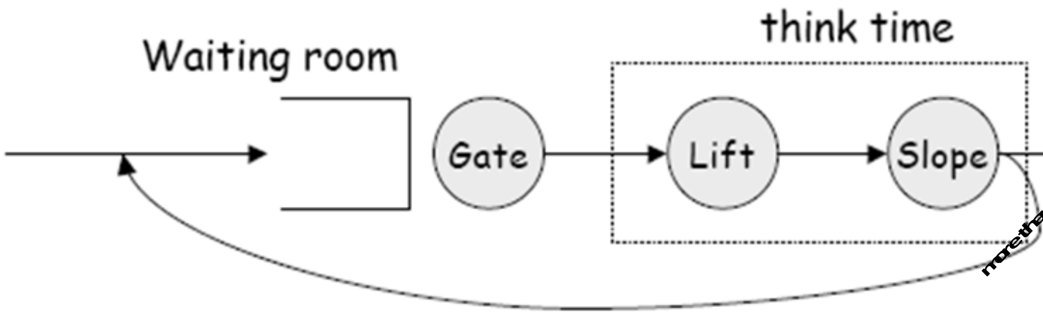
Second, the average response time R_0 of an arbitrary customer, conditional to its service time S_0 satisfies [47]

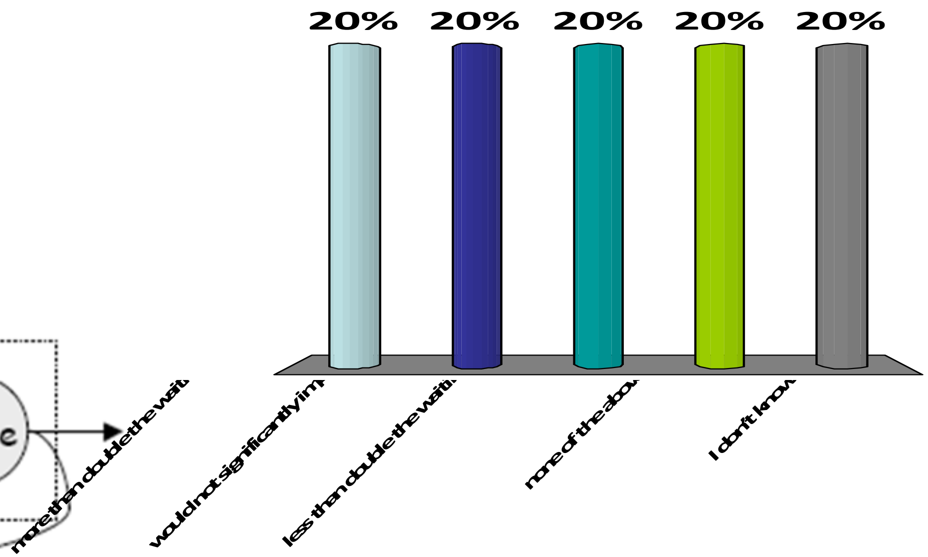
$$\mathbb{E}^0\left(R_0 \,|S_0 = x\right) = \frac{x}{1-\rho} \tag{8.19}$$

N is the number of skiers present (in average).

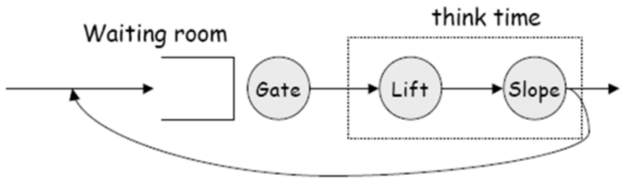
Doubling N would…

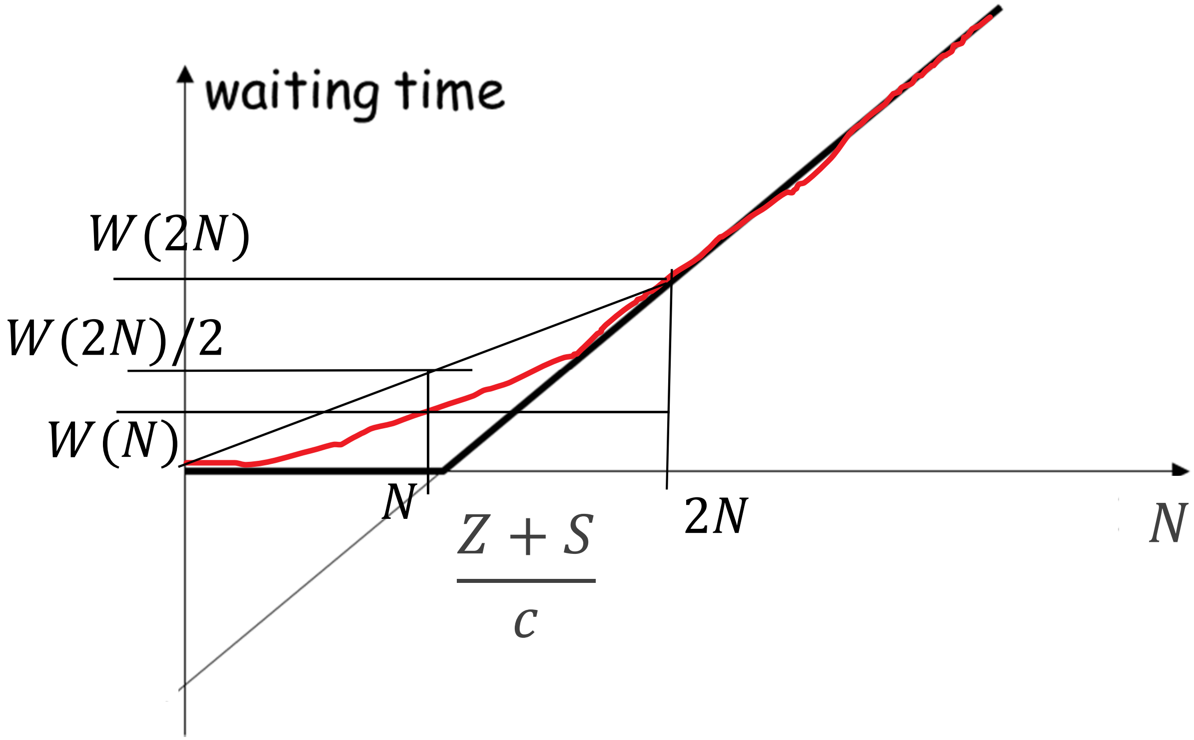
A. more than double the waiting time
B. would not significantly impact the waiting time
C. less than double the waiting time
D. none of the above
E. I don’t know





Solution





Z = avg think time, S = service time

$\lambda(W + Z + S) = N$

$W = \frac{N}{\lambda} - Z - S$

Congested system: $\lambda \approx c$

$$W \approx \frac{N}{c} - Z - S$$

Lightly loaded system: $W \approx 0$

The curve is likely to be convex $\Rightarrow$ AnswerA