Bonus Tests

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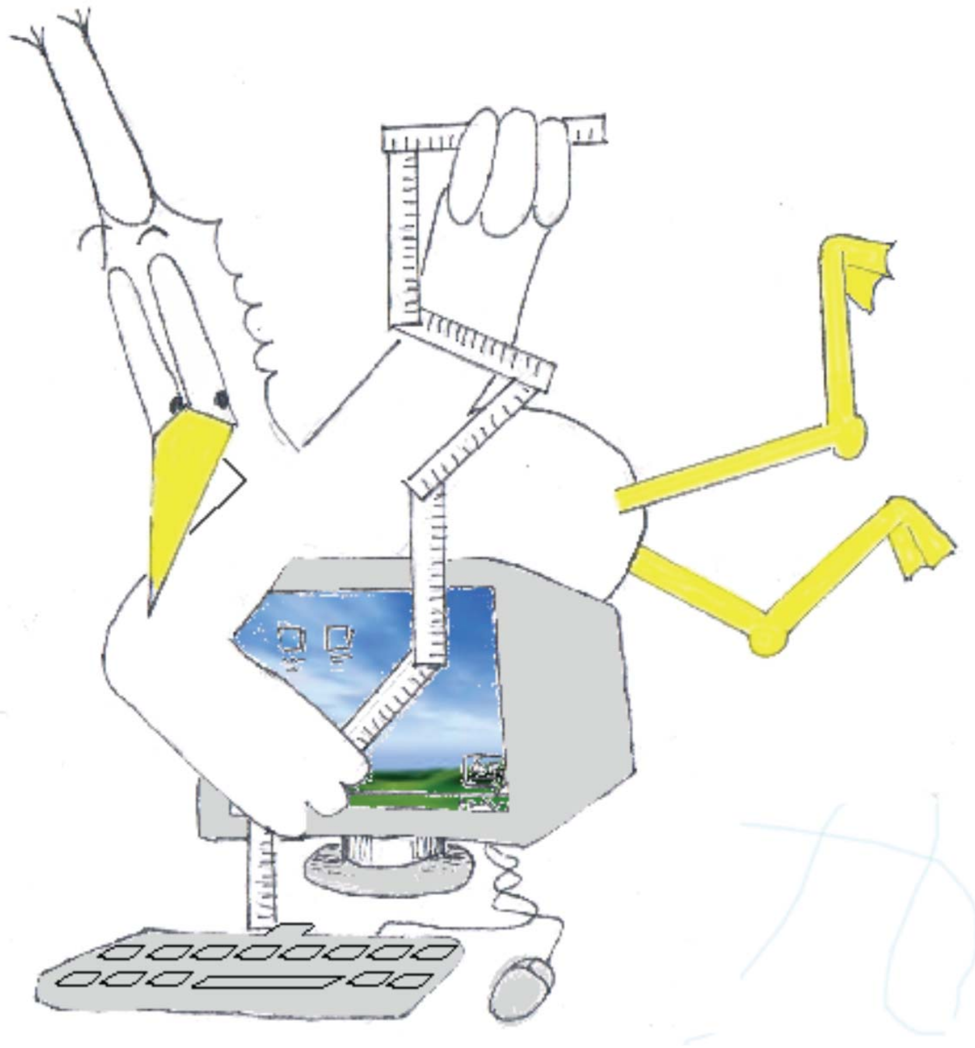


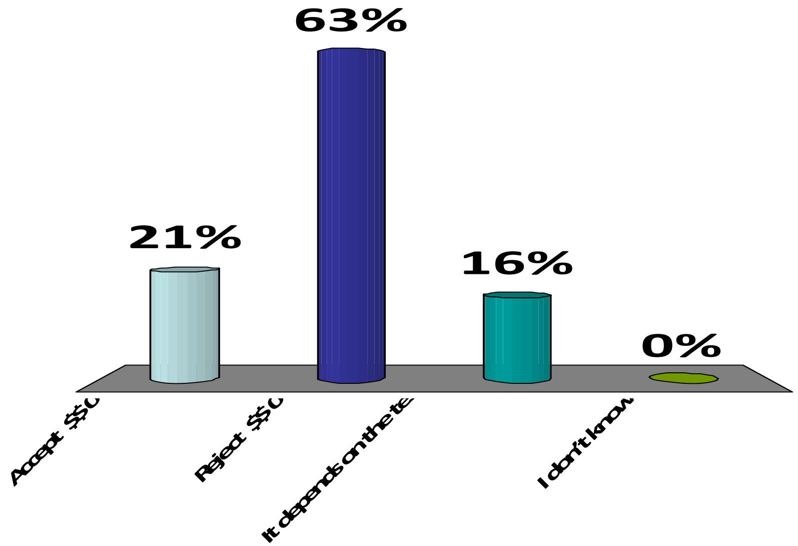
If the data is in the critical region we...

A. Accept H_0

B. Reject H_0

C. It depends on the test

D. I don't know



Solution

Answer B, by definition of the critical region

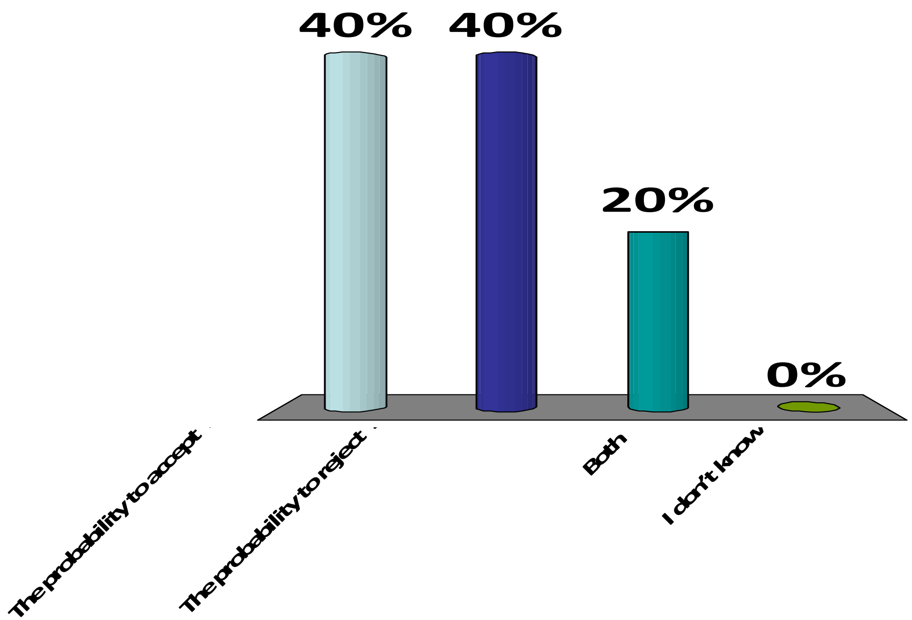
Saying that a test is of size 5% means that…

A. The probability to accept H_0 when it does not hold is ≤ 0.05

B. The probability to reject H_0 when it holds is ≤ 0.05

C. Both

D. I don't know



Solution

Answer B, by definition of the size of a test.

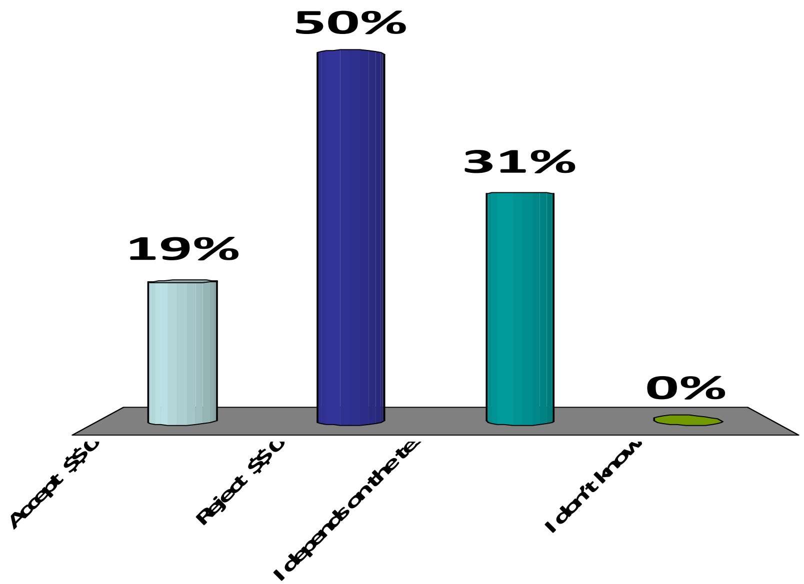
If the p −value of a test is small we…

A. Accept H_0

B. Reject H_0

C. I depends on the test

D. I don't know

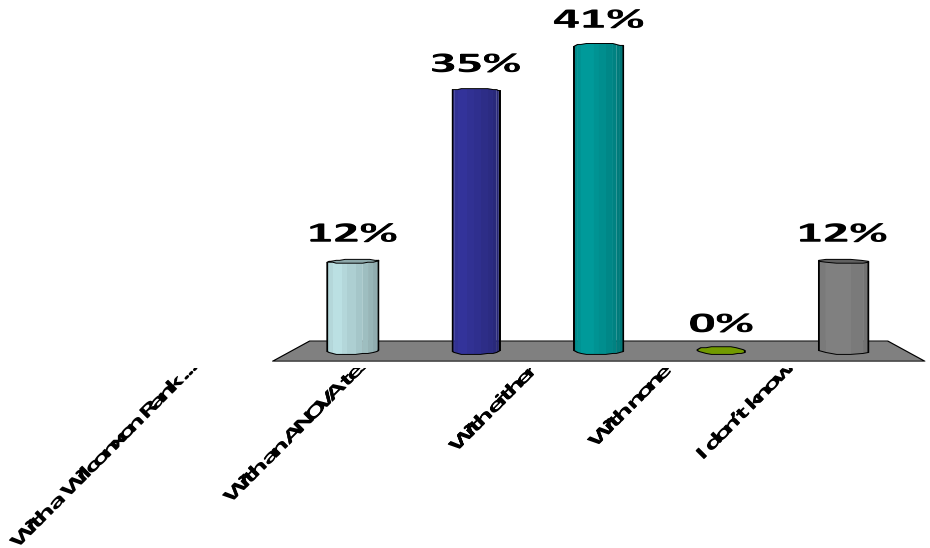


Solution

Answer B, by definition of the p-value

We have a collection of random variables X_i, Y_i which correspond to non paired simulation results with configuration 1 or 2. How can you test whether the configuration plays a role or not ?

A. With a Wilcoxon Rank Sum test
B. With an ANOVA test
C. With either
D. With none
E. I don’t know



Solution

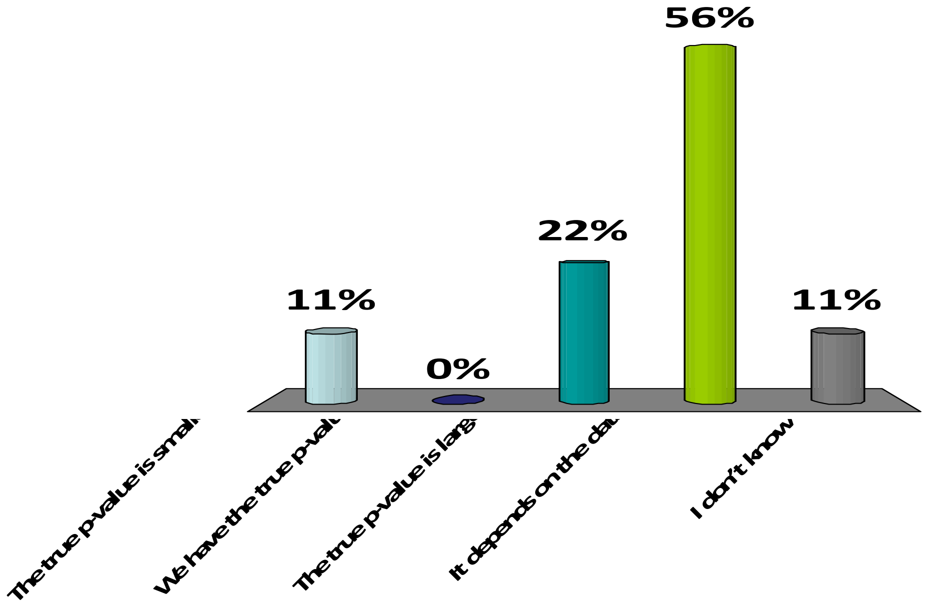
Answer C

A is robust and can be used if we can ensure that the simulation runs are independent and that the two distributions differ by a location shift, i.e. have same variance

B is applicable if, in addition, X_i and Y_i can be assumed gaussian with same variance

We test whether a distribution is gaussian using a Kolmogorov-Smirnov test against the fitted distribution. We obtain a p −value

A. The true p-value is smaller
B. We have the true p-value
C. The true p-value is larger
D. It depends on the data
E. I don’t know



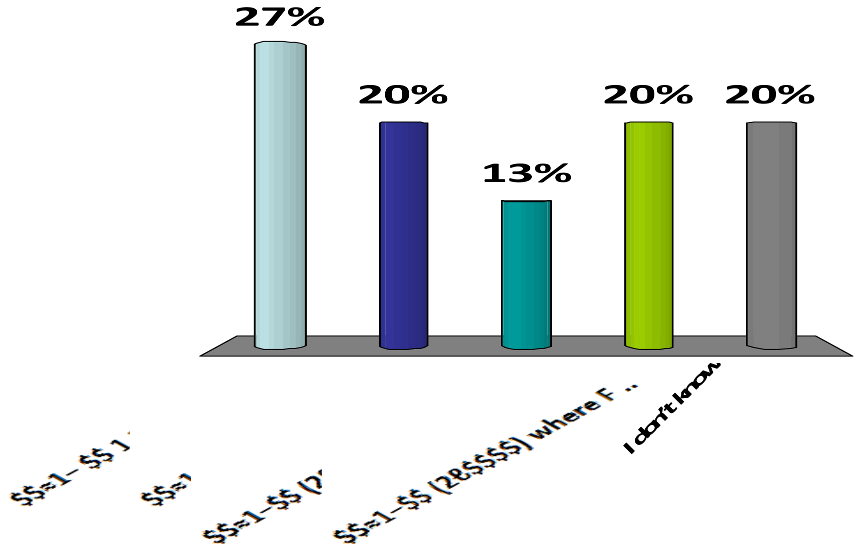
Solution

Answer A

The KS test applies if we are testing against a fixed, non fitted distribution F. By using a fitted distribution, we are biasing the test, we are making it more likely than should be to accept the distribution F, i.e. to accept H_0. The p-value is higher when we accept H_0 , i.e. we are overestimating the p-value.

We have two data sets X_i and Y_j believed to be iid and from one exponential distribution each. We test whether they come from the same distribution and make a likelihood ratio test. The log likelihood ratio statistic is lrs. The p-value is…

A. $p \approx 1 - \chi_1^2(2\ell rs)$
B. $p \approx 1 - \chi_2^2(2\ell rs)$
C. $p \approx 1 - F\,(2\ell rs)$ where F is the CDF of the standard exponential distribution
D. $p \approx 1 - F\,(2\ell rs)$ where F is the CDF of the standard Laplace distribution
E. I don't know



Solution

1. The log-likelihood is $m\log\lambda - \lambda\sum_i x_i + n\log\mu - \mu\sum_j y_j$ where $i = 1\ldots m, j = 1\ldots n$, λ [resp. μ] is the parameter of the expo distrib of X_i [resp. Y_j]

2. Under H_0: $\lambda = \mu$, max is for $\lambda^{-1} = \mu^{-1} = \frac{\sum_i x_i + \sum_j y_j}{m+n}$

$$\ell_0 = (m+n)\log\left(\frac{m+n}{\sum_i x_i + \sum_j y_j}\right) - m - n$$

3. Under H_1, max is for $\lambda^{-1} = \frac{\sum_i x_i}{m}$, $\mu^{-1} = \frac{\sum_j y_j}{n}$

$$\ell_1 = m\log\left(\frac{m}{\sum_i x_i}\right) + n\log\left(\frac{n}{\sum_j y_j}\right) - m - n$$

$$lrs = \ell_1 - \ell_0$$

4. The order is $p = 2 - 1 = 1$, $p \approx 1 - \chi_1^2(2\ell rs)$
Answer A