Forecasting Bonus Exercise

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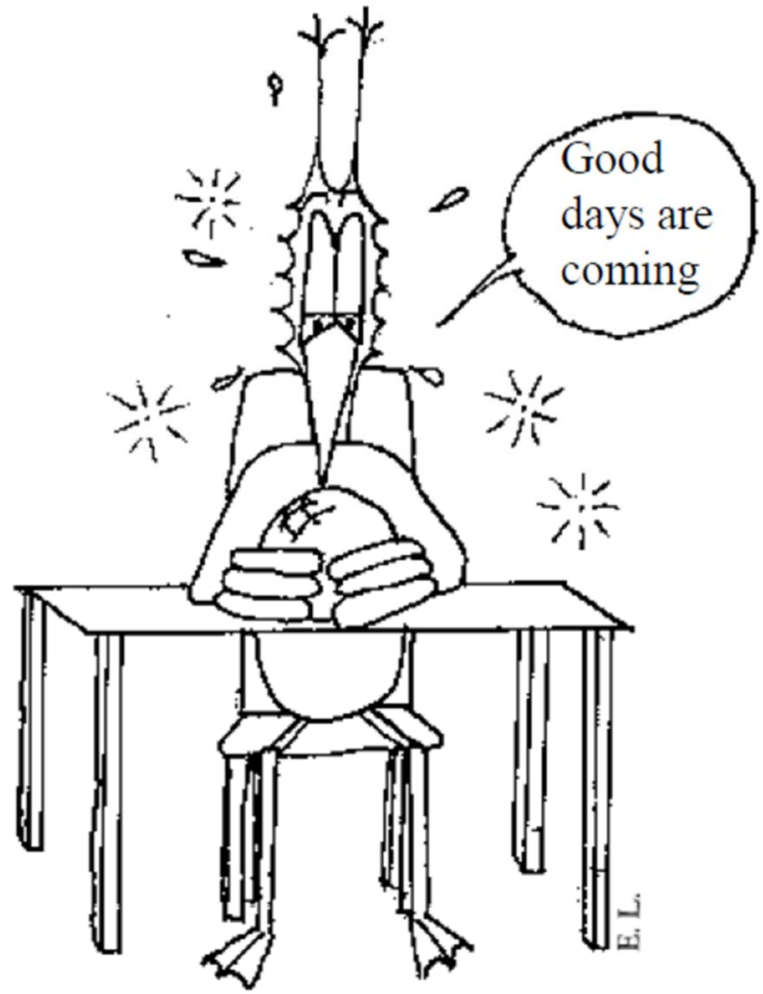


Exercise: Compute the prediction

We have a times series Y_t . We computed the differenced time series $X_t = Y_t - Y_{t-1}$ and found that X_t can be modelled as an AR process: $X_t = \epsilon_t + 0.5\, X_{t-1}$ where $\epsilon_t \sim$ iid $N(0, \sigma^2)$

1. Is this a valid ARIMA model ?
2. Compute a point forecast $\hat{X}_t(2)$
3. Compute a point forecast $\hat{Y}_t(2)$
4. Compute the first 3 terms of the impulse response of the filter $\epsilon \rightarrow Y$
5. Compute a prediction interval for Y_{t+2} done at time t
6. How would you compute a prediction interval using the boostrap ?

1. Is this a valid ARIMA model ?

A. Yes because X=$F\epsilon$ and F is an ARMA filter

B. Yes because $X = F\epsilon$ and F is a stable ARMA filter with stable inverse

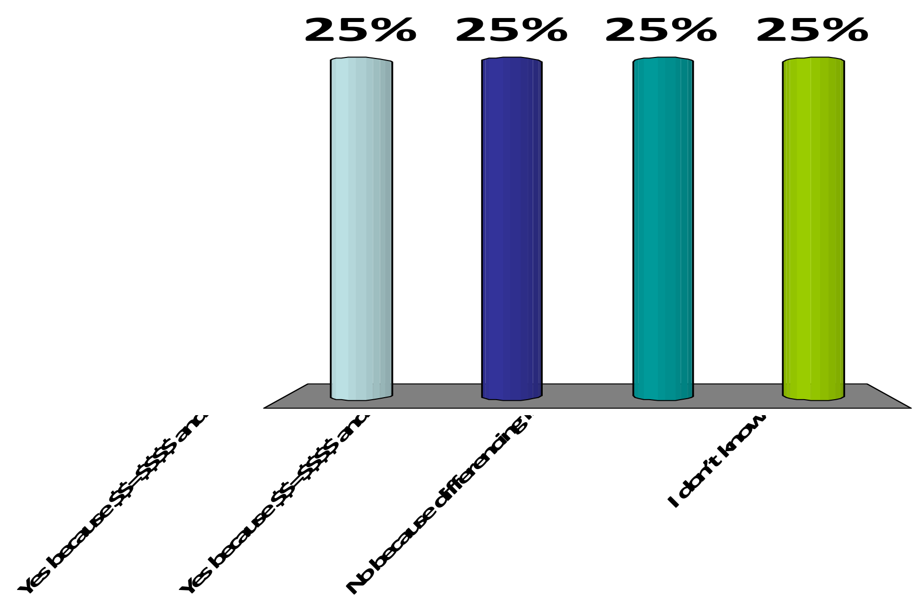
C. No because differencing is not a stable filter

D. I don't know

Exercise: Compute the prediction

We have a times series Y_t . We computed the differenced time series $X_t = Y_t - Y_{t-1}$ and found that X_t can be modelled as an AR process: $X_t = \epsilon_t + 0.5\ X_{t-1}$ where $\epsilon_t \sim$ iid $N(0,\sigma^2)$

1. Is this a valid ARIMA model ?
2. Compute the prediction formulae



2. The point predictions for X are…

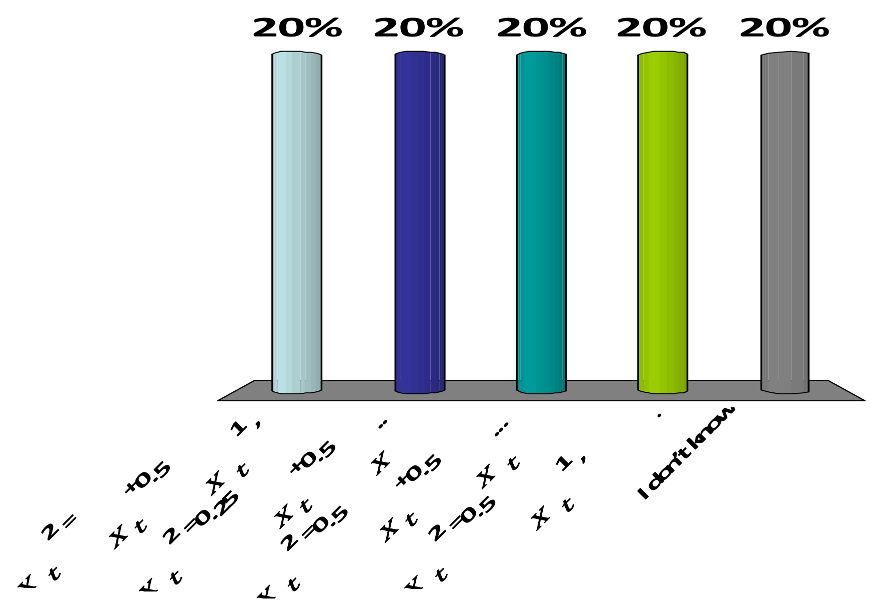
A. $\hat{X}_t(2) = X_t + 0.5\hat{X}_t(1),\ \ \hat{X}_t(1) = 0.5X_t$

B. $\hat{X}_t(2) = 0.25X_t + 0.5\hat{X}_t(1),\ \ \hat{X}_t(1) = 0.5X_t$

C. $\hat{X}_t(2) = 0.5X_t + 0.5\hat{X}_t(1),\ \ \hat{X}_t(1) = -0.5X_t$

D. $\hat{X}_t(2) = 0.5\hat{X}_t(1),\ \ \hat{X}_t(1) = 0.5X_t$

E. I don't know



3. The point predictions for Y are …

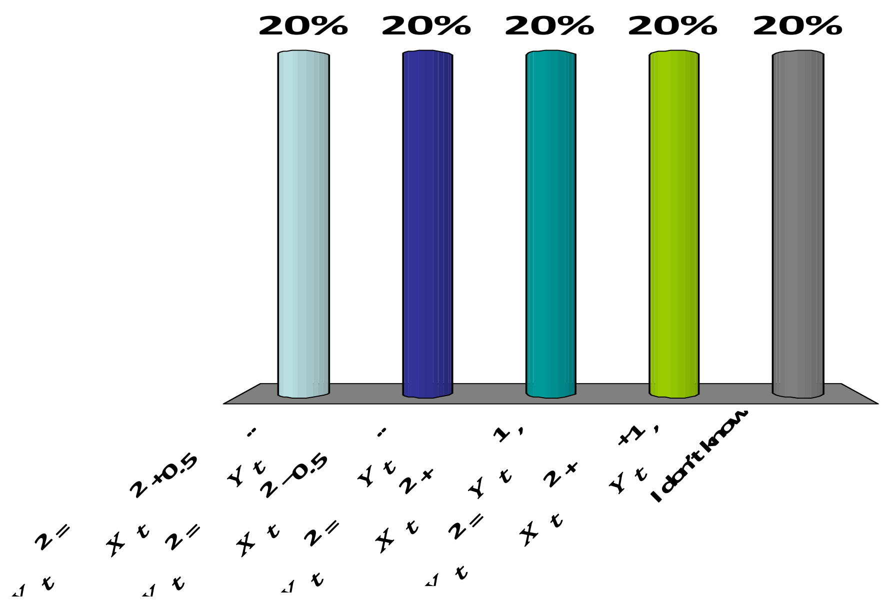
A. $\hat{Y}_t(2) = \hat{X}_t(2) + 0.5Y_t(1),\ \hat{Y}_t(1) = \hat{X}_t(1) + 0.5Y_t$

B. $\hat{Y}_t(2) = \hat{X}_t(2) - 0.5Y_t(1),\ \hat{Y}_t(1) = \hat{X}_t(1) - 0.5Y_t$

C. $\hat{Y}_t(2) = \hat{X}_t(2) + \hat{Y}_t(1),\ \hat{Y}_t(1) = \hat{X}_t(1) + Y_t$

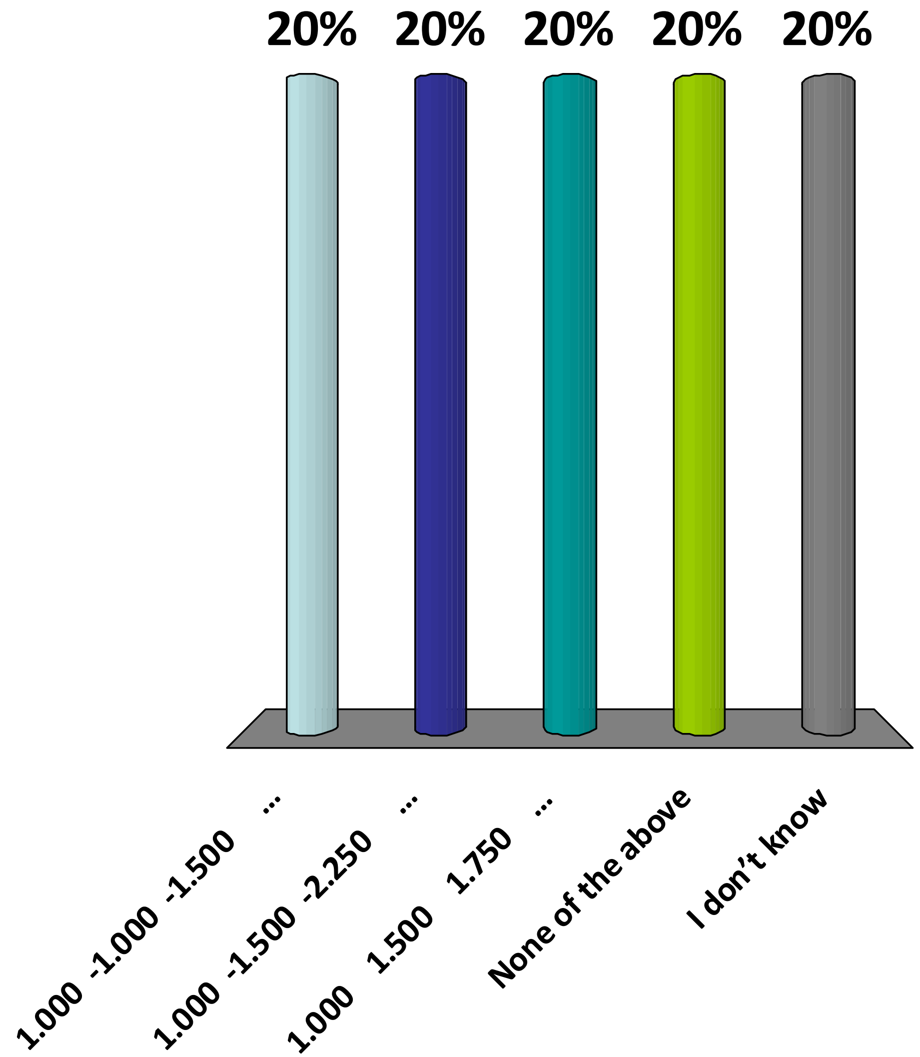
D. $\hat{Y}_t(2) = \hat{X}_t(2) + Y_{t+1},\ \hat{Y}_t(1) = \hat{X}_t(1) + Y_t$

E. I don't know



4. What is the impulse response of the filter $\epsilon \to Y$?

A. 1.000 -1.000 -1.500 …
B. 1.000 -1.500 -2.250 …
C. 1.000 1.500 1.750 …
D. None of the above
E. I don't know



5. A prediction interval for Y_{t+2} done at t is …

A. $\hat{Y}_t(2) \pm 1.96 \times \sigma$

B. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{1.25}\sigma$

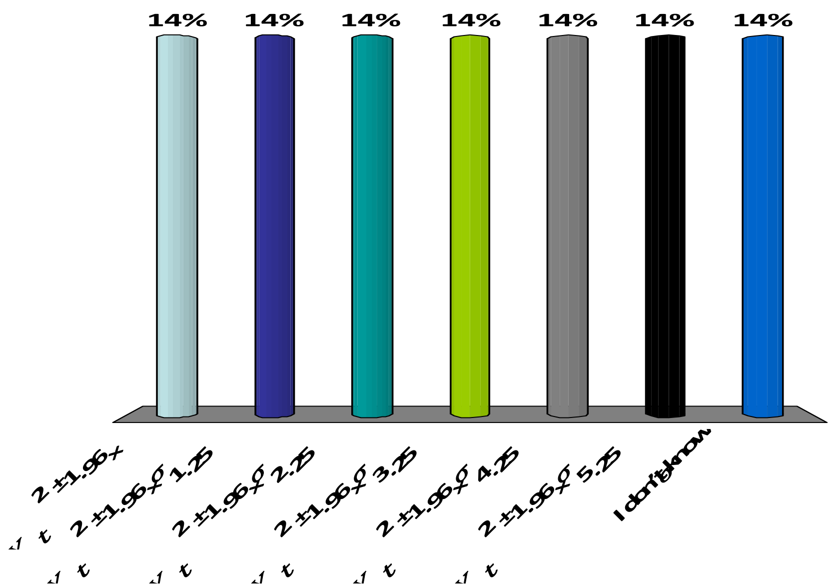
C. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{2.25}\sigma$

D. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{3.25}\sigma$

E. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{4.25}\sigma$

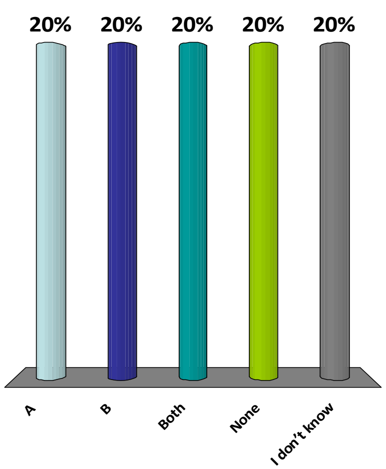
F. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{5.25}\sigma$

G. I don't know



6. Which is a correct implementation of the bootstrap for computing 95%-prediction intervals at time t and lag 2 ?

A. A
B. B
C. Both
D. None
E. I don't know



A

Compute the time series $\epsilon_s = X_s - 0.5X_{s-1}$, $s = 3{:}\,t$
do $r = 1{:}\,999$ {
draw $e_s^r, s = 3{:}\,(t+2)$ with replacements from ϵ_t
compute $X_{1:t}^r, Y_{1:t}^r$ and $\hat{Y}_t^r(2)$ using $X_s^r = e_s^r + 0.5X_{s-1}^r, Y_s^r = X_s^r + Y_{s-1}^r$ and the formula for $\hat{Y}_t^r(2)$
$Y_{t+2}^r = e_{t+2}^r + 1.5e_{t+1}^r + \hat{Y}_t^r(2)$}
Prediction interval is $[Y_{t+\ell}^{(25)}, Y_{t+\ell}^{(975)}]$

B

Compute the time series $\epsilon_s = X_s - 0.5X_{s-1}$, $s = 3{:}\,t$
do $r = 1{:}\,999$ {
draw e_1^r, e_2^r with replacements from ϵ_t
$Y_{t+2}^r = e_1^r + 1.5e_2^r + \hat{Y}_t(2)$}
Prediction interval is $[Y_{t+\ell}^{(25)}, Y_{t+\ell}^{(975)}]$